QUIPU AS209

Museum identification: HP519 (Royal Ontario Museum, Toronto)

Main cord: color LB

b[1] 26.5 cm: group of 8 pendant cords (1-8), then space of 1.5 cm.

30.0 cm: group of 9 pendant cords (9-17), then space of 10.0 cm.

42.0 cm: group of 9 pendant cords (18-26), then space of 1.0 cm.

45.0 cm: group of 9 pendant cords (27-35), then a single pendant cord (36), then space of 1.0 cm.

48.5 cm: group of 11 pendant cords (37-47), then space of 0.5 cm.

51.5 cm: group of 9 pendant cords (48-56), then space of 5.5 cm.

59.0 cm: end ¢

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
1	1s(7.0);3s(15.0);8L(21.0)	49.5¢	DB	138	2:12.0
1s1	1s(3.0);3L(9.5)	26.0¢	DB	13	
1s2	2L(8.5)	15.0¢	DB	2	
2	1s(7.0);2s(15.5);3L(21.5)	43.0¢	DB	123	2:12.5
2s1[2]	-	0.0b	DB	?	
2s2	1E(10.0)	31.5¢	DB	1	
3	2s(7.5);1s(15.0);5L(23.5)	46.5¢	DB	215	3:12.5-13.0
3s1[2]	-	0.0b	DB	?	
3s2	2s(2.5);5L(8.5)	17.5¢	DB	25	
3s3	2L(8.5)	29.0¢	DB	2	

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
4	1s(7.0);8s(16.0)	40.0¢	DB	180	2:12.5
4s1	2s(2.5);6L(8.5)	16.5¢	DB	26	
4s2	1E(9.5)	28.5¢	DB	1	
5	1s(7.0);8s(15.5);6L(21.5)	30.5¢	DB	186	1:11.5
5s1	1s(2.5);9L(8.5)	13.5¢	DB	19	
6	4s(7.5);2s(15.5);1E(21.5)	41.5¢	DB	421	2:12.5
6s1	8s(3.5);5L(8.5)	11.5¢	DB	85	
6s2	2L(8.5)	29.5¢	LB	2	
7	4s(7.5);8s(17.0);3L(22.5)	37.5¢	DB	483	5:12.0-13.0
7s1	-	32.0¢	KG	0	
7s2	3s(3.0);4L(9.5)	15.5¢	DB	34	
7s3	2L(9.0)	30.0¢	DB	2	
7s4[2a]	-	1.5b	DB	?	
7s5	-	6.0b	DB:KB	?	
8	5s(7.5);5s(14.0);5L(21.0)	40.0¢	DB	555	3:11.0-11.5
8s1	1s(1.0);1s(4.0)	16.0¢	LB-GY	110	
8s2	2s(3.0);1E(8.0)	20.0¢	DB	21	
8s3	5s(4.0)	33.0¢	DB	50	
9	1s(7.0);2s(14.0);8L(20.5)	33.5¢	W	128	2:9.0
9s1	4L(10.0)	18.0¢	W	4	
9s2	2s(4.5);2L(11.0)	21.0¢	DB	22	
10	1s(7.0);5s(14.5);8L(21.0)	37.0¢	W	158	2:9.0
10s1	2L(10.5)	24.0¢	DB	2	
10s2	2s(4.5);1E(11.5)	20.5¢	DB	21	

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
11	2s(7.0);6s(14.0);7L(21.0)	36.0	W	267	2:9.5
11s1	2L(9.5)	14.0¢	DB	2	
11s2	2s(4.0);8L(11.5)	16.0¢	DB	28	
12	3s(6.5);2s(13.0);3L(20.0)	42.0¢	W	323	1:9.0
12s1	2s(4.0);8L(11.0)	16.0b	DB	28	
13	2s(7.0);2s(14.0);1E(21.5)	47.5¢	W	221	2:10.0
13s1	-	1.5b	W	?	
13s2	2s(3.5);2L(11.5)	22.5¢	DB	22	
14	1s(7.0);7s(14.5);8L(21.5)	30.5¢	W	178	1:9.5
14s1	2s(4.5);6L(12.0)	20.0¢	DB	26	
15	2s(7.5);9s(15.5);6L(21.5)	31.0¢	W	296	2:10.0
15s1	2L(12.0)	25.0¢	W	2	
15s2	4s(4.5);1E(11.5)	19.5¢	DB	41	
16	1s(7.0);8s(16.5);8L(23.5)	36.5¢	DB	188	1:11.0
16s1	1s(3.5)	23.5¢	DB	10	
17[2b]	1s(2.5);2s(8.0); 1s(16.0);6L(23.5)	? ¢	W	1216	3:12.0-12.5
17s1	5s(4.5);7L(11.0)	18.0¢	W	57	
17s2	1s(0.5);2s(3.5);4L(11.0)	21.0	DB	124	
17s3	-	0.5b	KB	?	
18	-	49.0¢	LB	0	
19	5s(9.0);4s(13.0);9L(20.5)	25.0¢	LB	549	1:9.5
19s1	2L(10.5)	36.5¢	DB	2	
20	5s(9.0);4s(12.5);9L(19.0)	24.0¢	LB	549	1:9.5
20s1	1s(2.5)	37.5¢	DB	10	

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
21	7s(9.0);1E(19.5)	36.5¢	LB	701	1:9.5
21s1	1s(3.0);9L(10.0)	23.0¢	DB	19	
22	4s(9.0);2s(13.5);1E(19.5)	40.5¢	LB	421	3:9.0-9.5
22s1	2L(9.0)	38.0¢	DB	2	
22s2	1E(9.5)	29.5¢	OG:GY	1	
22s3	2s(0.5);5s(4.0);7L(9.5)	16.0¢	LB:W	257	
23	3s(9.0);7s(13.5);4L(19.5)	30.5¢	LB	374	2:9.5
23s1	4L(9.5)	25.5¢	DB	4	
23s2	2s(1.0);7s(4.0)	24.0¢	LB:W	270	
24	1s(3.0);7s(9.0);8s(14.0)	29.0¢	LB	1780	1:10.5
24s1	1s(3.0);4L(10.0)	27.0¢	DB	14	
25	9s(9.0);7s(15.0);7L(19.5)	22.0¢	LB	977	4:11.0-11.5
25s1	4L(7.0)	29.0¢	DB	4	
25s2	2s(1.0);6s(4.0);8L(8.0)	21.0¢	LB	268	
25s3	2s(3.0)	30.0¢	LB	20	
25s4	5L(7.5)	17.5¢	KB:W	5	
26	2s(3.0);4s(8.5); 9s(14.5);3L(20.5)	29.5¢	LB	2493	1:10.5
26s1	1s(3.0);5L(10.0)	26.0¢	DB	15	
27	7s(8.0);7s(14.5);5L(20.5)	27.5¢	W	775	
28	4s(8.0);4s(14.5);2L(21.0)	40.0	W	442	
29	1s(2.5);1s(7.0); 1s(13.5);1E(20.0)	41.0¢	W	1111	
30	1s(2.5);8s(14.5);3L(20.0)	40.0¢	W	1083	
31	7s(7.5);3s(14.5);6L(20.0)	33.5¢	W	736	1:10.5

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
31s1[3]	-	0.5b	KB	?	
32	6s(8.0);8s(14.5)	40.5¢	W	680	1:11.0
32s1[3]	3s(3.0)	7.5b	KB	3?	
33	8s(8.0);8s(14.0);7L(19.5)	26.5¢	W	887	3:11.0-11.5
33s1[3]	-	0.0b	KB	?	
33s2	4s(3.0);8L(8.0)	11.0¢	DB	48	
33s3	5s(3.0);2L(8.0)	21.0¢	DB:KB	52	
34	5s(8.0);4s(14.5)	35.5¢	W	540	
35	2s(3.0);2s(8.0); 6s(15.0);5L(21.0)	48.0¢	W	2265	5:11.5-12.0
35s1[3]	2s(1.0);6s(4.0)	7.0b	KB	26?	
35s2	7L(7.5)	12.0b	LB:W	7	
35s3	7s(3.0);3L(7.0)	17.0¢	DB:KB	73	
35s4	4L(7.5)	20.0¢	DB	4	
35s5	7L(7.5)	13.5¢	OG:GY	7	
36	1s(2.5);7s(8.5); 7s(14.5);6L(19.5)	26.0¢	LB	1776	1:11.0
36s1	7L(9.0)	37.0¢	DB:KB	7	
37	8s(9.5);4L(21.0)	36.0¢	LB	804	1:13.5
37s1	2L(7.0)	42.0¢	LB	2	
38	8s(9.5);1s(15.5)	53.5¢	LB	810	2:12.0
38s1	2s(4.0);3L(10.0)	17.5¢	OG	23	
38s2	1s(3.0)	34.0¢	LB	10	
39	1s(3.5);7s(17.0);6L(22.5)	45.5¢	LB	1076	2:12.5
39s1	3s(3.0);8L(9.0)	12.5¢	OG	38	

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
39s2	1s(3.0);9L(9.5)	23.5¢	LB	19	
40	1s(3.0);1s(8.0); 3s(15.0);7L(20.5)	47.5¢	LB	1137	3:11.5-12.0
40s1	1s(3.0);7L(8.0)	14.0¢	OG	17	
40s2	2L(8.0)	31.0¢	LB	2	
40s3	2L(7.5)	22.5¢	GY	2	
41	9s(9.0);6s(15.5);6L(21.0)	36.0¢	LB	966	3:12.5-13.0
41s1	6L(8.0)	17.0¢	OG	6	
41s2	4L(6.5)	28.5¢	LB	4	
41s3	1E(8.0)	21.0¢	GY	1	
42	2s(3.0);5s(8.0); 9s(16.0);3L(20.5)	35.5¢	LB	2593	2:12.0
42s1	8s(3.5);2L(8.0)	10.5¢	OG	82	
42s2	1s(2.5);4L(8.0)	28.0¢	LB	14	
43	1s(3.0);8s(8.5); 3s(15.5);3L(20.5)	50.5¢	LB	1833	3:12.0-12.5
43s1	3s(3.0);3L(8.0)	12.5¢	OG	33	
43s2	4L(7.5)	29.5¢	LB	4	
43s3	2s(3.0);7L(8.0)	17.5¢	LB	27	
44	1s(7.0);7s(15.0)	34.0¢	LB	170	3:11.0-11.5
44s1	1s(3.0)	19.0¢	OG-GY	10	
44s2	6L(8.5)	24.5¢	DB:KB	6	
44s3	1s(3.0);8L(8.5)	18.5	LB	8	
45	4s(15.0);5L(19.5)	34.5¢	LB	45	1:11.0
45s1	7L(8.0)	39.0¢	DB	7	
46	3s(14.0);4L(18.5)	41.5¢	LB	34	

Cord	Knots (no.,type,position)	Length	Color	Value	Subsidiaries (no.,position)
47	4s(2.5);7s(14.5);4L(19.5)	53.5¢	LB	4074	2:10.5
47s1	4s(3.0);1E(8.0)	15.0¢	OG	41	
47s2	1s(3.0);5L(8.5)	29.5¢	LB	15	
48	1s(2.5);5s(14.5);7L(19.0)	41.0¢	W	1057	
49	7s(9.0);5s(15.0);7L(20.0)	39.0¢	W	757	
50	1s(2.5);6s(9.0); 3s(15.0);5L(20.5)	44.5¢	W	1635	
51	1s(2.5);5s(8.0); 9s(14.5);7L(20.0)	31.0¢	W	1597	
52	1s(2.5);1s(7.5) 4s(14.0);9L(19.5)	40.5¢	W	1149	
53	1s(2.5);4s(14.0);3L(20.0)	54.0¢	W	1043	
54	1s(2.5);5s(8.0); 3s(14.0);2L(19.5)	53.5¢	W	1532	1:11.0
54s1[4]	-	2.0b	KB	?	
55	8s(8.5);5s(14.5);6L(20.0)	41.0¢	W	856	1:11.5
55s1[4]	-	0.5b	KB	?	
56	6s(2.5);5s(8.0); 6s(14.0);9L(20.5)	29.0¢	W	6569	2:11.0
56s1	3s(2.0);3L(8.0)	20.0¢	KB	33	
56s2	1s(2.0)	28.0b	DB:KB	10	

Observations

1. Construction Note: The end of the main cord has been cut or cut and wrapped. This finishing may be intentional and the quipu complete. At 19.5 cm the main cord has been repaired or joined to another piece of the same cord.

2. **a)** Two cord fragments were found associated with the first group of pendants. They are both DB. One is broken at both ends with a cluster of 2s at 1.0 cm from one end and an overall length of 3.5 cm. The other has an overall length of 38.5 cm. Beginning at its broken end, there is a cluster of 2s at 2.5 cm from the end then a space of 6.0 cm until a 1E and then 30.0 cm until a finished end. These are probably parts of 2s1 or 3s1 or 7s4.

   b) This pendant is broken at 12.5 cm. However, this is the only place the fragment could have come from and so is assumed to be part of this pendant.

3. A cord fragment was found associated with the group of pendants 27-35. The fragment is color KB. It is broken at both ends with a knot cluster of 4L at 1.5 cm from one end. Its overall length is 17.5 cm. It is probably a part of 31s1, 32s1, 33s1, or 35s1.

4. Three cord fragments were found associated with the last group of pendants. All are color KB. One is just a cluster of 3s. Another is a cluster of 9L and then 2.0 cm. Both of these are broken at both ends. The last is 5.0 cm with one broken end and one finished end. These are probably parts of 54s1 or 55s1.

5. By space, color and magnitude of numbers, the quipu is separated into two parts. Part I is the first 17 pendants forming two groups by spacing and color; group 1 is 8 pendants with color DB and group 2 is 9 pendants of color W. The pendant values range from 123 to 555 (and one value in

the 1200's). Part II contains four groups by spacing and color. The group colors are alternately LB and W. Part II begins with a blank cord (P18) associated with group 3 but much longer than the pendants that follow it. We consider it to be serving as a marker reinforcing the separation from Part I. Nine pendants in group 4 are color W but a tenth (P36), slightly apart from the others, is of color LB. With the exception of three pendants (P44, P45, P46) whose values are 170 and below, the pendant values in Part II are 374 to 2593 (and two values above 4000). P44, P45, and P46 will be seen to be anomalous for other reasons as well. Overall, the structure is

Part I: Group 1 - an 8 pendant group of color DB

Group 2 - a 9 pendant group of color W

Part II: Marker

Group 3 - an 8 pendant group of color LB

Group 4 - a 9 pendant group of color W, then an LB pendant.

Group 5 - an 8 pendant group of color LB plus three anomalous pendants between the 7th and 8th pendants in the group.

Group 6 - a 9 pendant group of color W.

These can be referred to as P_{ij} where $i = 1,\ldots,6$. For $i = 2,4,6$, $j = 1,\ldots,9$; for $i = 1,3,5$, $j = 1,\ldots,8$ (and for $i = 5$, also $j = 1',5',6',7'$).

6. **a)** Each P_{ij} has one or more subsidiary for i = 1,3,5 and j = 1,...,8.

   i) Each P_{ij} has at least one DB subsidiary.

   ii) The first subsidiary on each P_{3j} is color DB.

   iii) Each P_{5j} (j = 1,...,8) has at least one LB subsidiary. When there are two or three subsidiaries, the color order is consistently some or all of OG, LB, LB or GY.

   iv) For j = 1,...,8: value on DB subsidiary on P_{3j} = value on first LB subsidiary on P_{5j}. (Note that P_{5j}, j = 5',6',7' are omitted from these patterns).

   b) Each P_{2j} has at least one DB subsidiary. (Note: Combining this with comment 6ai above, each pendant in Part I has at least one DB subsidiary.) Where there are two or three subsidiaries, the color order is consistently some or all of W, DB, DB or KB. Each P_{ij} for i = 4,6 and j = 1,2,3,4 has no subsidiaries.

7. **a)** $P_{ij} < P_{3j} < P_{5j}$ for j = 1,...,8.

   b) $P_{2j} < P_{4j} < P_{6j}$ for j = 1,...,9.

8. Within Part II:

   a) The pendant values plus their subsidiary values have the same rank order in groups 3 and 5 except for a reversal between pendants 3 and 4. That is,

$$\left.\begin{array}{l} P_{i8} > P_{i6} > P_{i7} > P_{i4} > P_{i5} > P_{i2} > P_{i1} \\ P_{i8} > P_{i6} > P_{i7} > P_{i3} > P_{i5} > P_{i2} > P_{i1} \end{array}\right\} \text{ for } i = 3,5 \text{ (with subsidiaries)}$$

Also, with or without subsidiaries

$$\left.\begin{array}{l} P_{i8} > P_{i6} > P_{i7} > P_{i4} > P_{i5} \\ P_{i8} > P_{i6} > P_{i7} > P_{i3} > P_{i2} \geq P_{i1} \end{array}\right\} \quad \text{for } i = 3,5$$

b) The pendant values have the same rank order in groups 4 and 6 except for a reversal between pendants 1 and 5. That is,

$$\left.\begin{array}{l} P_{i9} > P_{i3} > P_{i4} > P_{i7} > P_{i5} > P_{i6} > P_{i8} > P_{i2} \\ P_{i9} > P_{i3} > P_{i4} > P_{i7} > P_{i1} > P_{i6} > P_{i8} > P_{i2} \end{array}\right\} \quad \text{for } i = 4,6$$

c) For all $j = 1,\ldots,8$ with subsidiaries P_{5j}/P_{3j} is between 1.46 and 1.70. The ratios for $j = 2,5,6,7$ with subsidiaries are very close to each other and very close to 3/2: That is,

$$P_{5j}/P_{3j} = 1.5 \pm .7\% \qquad \text{for } j = 2,5,6,7 \text{ (with subsidiaries)}$$

Also, the ratios for $j = 2,5$ differ from each other by only .02%.

d) For $j = 1,\ldots,8$ P_{6j}/P_{4j} is between 1.36 and 1.73. (For the exception $j = 9$, $P_{69}/P_{49} = 2.900$.) The ratios for $j = 3,4$ are very close to each other, namely both are $1.4731 \pm .1\%$.

9. Within group 5, the ratios P_{56}/P_{57} and P_{54}/P_{51} are the same to within .03%. This is of interest because they are remarkably close to $\sqrt{2}$. In fact, $P_{54}/P_{51} = 379/268 = \sqrt{2} - .0024\%$. There is no reason to hypothesize that the ratios of these pendant values or $\sqrt{2}$ were of any importance to the quipumaker. For us, however, it suggests an excellent approximation for $\sqrt{2}$.

10. **a)** The number of pendant values that are multiples of P_{56}' and P_{57}' suggest their values as some kind of units,

$P_{14} = 4\ P_{56}'$
$P_{48} = 12\ P_{56}' = 3\ P_{14}$
$P_{52} = 18\ P_{56}'$

$P_{56}' = 45$ so these are 4,12,18 times 45

and

$P_{55}' = 5\ P_{57}'$
$P_{35} = 11\ P_{57}'$
$P_{42} = 13\ P_{57}' = 2\ P_{25}$
$P_{46} = 4\ P_{55}' = 20\ P_{57}'$

$P_{57}' = 34$ so these are 5,11,13,20 times 34
These are also 2,10,22,26, 40 times 17 and, in addition $P_{25} = 13 \cdot 17$, $P_{24} = 19 \cdot 17$.

b) The number 17 is prominent in that it is a factor of about 12% of the pendant values. (It and two multiples of it, 34 and 85, also appear on subsidiaries but it is not prominent among subsidiary values.)

11. Sum cords:

a) The last pendant in each of groups 1,3,5 is related to the sum of the first four pendants in its group.

G1: P_{18} & its subsidiaries $\approx \sum_{j=1}^{4} P_{1j}$ & their subsidiaries
$(736 \approx 726+2$ broken subsidiaries$)$

G3: P_{38} & its subsidiaries $\approx \sum_{j=1}^{4} P_{3j}$ & their subsidiaries
$(2508 \approx 2511)$

G5: $P_{58} \approx \sum_{j=1}^{4} P_{5j} + \sum_{j=5'}^{7'} P_{5j}$ $\quad (4074 \approx 4076)$

b) The last pendant in group 6 is approximately equal to the sum of the last pendants in groups 3 and 5. By comment a) above, they in turn were also sums.

$P_{69} = P_{38} + P_{58} + 2.$

c) We have labeled the extra LB pendant just after group 4 as P_{51}' because by color and value it appears related to group 5. That is, $P_{51}' = P_{52} + P_{55}$.

12. Although there seems to be little consistency with position, many pendant values are the sums of two or more other pendant values. Some of the details are included here but, so far, an overall explanation or generalization is lacking.

a) There are altogether thirteen values that are sums of pairs of other values. They are P_{13}, P_{27}, P_{31}, P_{32}, P_{35}, P_{47}, P_{57} (in two ways), P_{51}', P_{61}, P_{64}, P_{65}, and P_{67} (in two ways).

b) Four values in groups 1 and 2 can be expressed as the sum of three other values. Then, with only five exceptions, each of the values in groups 3-6 can be expressed as the sum of three other values. Moreover, they have several such expressions. In group 2, one value can be expressed as the sum of three values in four different ways. In groups 3-6, two can be so expressed in two different ways, three in three different ways, one in four different ways, eight in five different ways, three in six different ways, four in seven different ways, and one each in eight, ten, and eleven different ways. Since the summands can in turn be sums, these can be combined to form longer sum chains. As one example we use P_{61} which has one expression as a sum of two values and five as a sum of three values. Because these involve sums, the sums can be extended to include six expressions as sums of four values, four expressions as sums

of five values, and one expression each as sums of six and seven values.

$$P_{61} = P_{55}' + P_{47} = P_{55}' + (P_{15} + P_{33}) = P_{55}' + P_{15} + (P_{11} + P_{27} + P_{23})$$

$$= P_{55}' + P_{15} + P_{11} + P_{27} + (P_{57}' + P_{56}' + P_{28})$$

$$= P_{55}' + (P_{22} + P_{14} + P_{31}) = P_{55}' + P_{22} + P_{14} + (P_{21} + P_{16})$$

$$= P_{55}' + (P_{12} + P_{13} + P_{31}) = P_{55}' + P_{12} + P_{13} + (P_{21} + P_{16})$$

$$= P_{55}' + (P_{56}' + P_{34} + P_{16}) = (P_{13}) + P_{34} + P_{16}$$

$$= P_{55}' + (P_{26} + P_{42} + P_{23}) = P_{55}' + P_{26} + P_{42} + (P_{57}' + P_{56}' + P_{28})$$

$$= P_{48} + (P_{57}' + P_{17}) = P_{48} + (P_{25} + P_{27}) = P_{48} + P_{25} + (P_{11} + P_{12})$$

$$= P_{48} + P_{25} + (P_{56}' + P_{12} + P_{21})$$

$$= P_{18} + P_{21} + P_{35} = P_{18} + P_{21} + (P_{16} + P_{28})$$

c) All values in groups 1 and 2 that are sums can be expressed such that the summands are only from groups 1,2, and 5' (those three anomalous pendants in group 5).

$P_{11} + P_{22} = P_{27}$; $P_{55}' + P_{56}' = P_{13}$; $P_{56}' + P_{57}' + P_{28} = P_{23}$;

$P_{12} + P_{21} + P_{5,4+j}' = P_{j,5+j}$ for $j = 1,2$;

$$P_{29} = P_{17} + P_{18} + P_{26} = P_{55}' + P_{12} + P_{15} + P_{21} + P_{26} + P_{28}$$

$$= P_{56}' + P_{18} + P_{21} + P_{23} + P_{25} = P_{56}' + P_{57}' + P_{11} + P_{13} + P_{23} + P_{25} + P_{27}$$

$$= P_{56}' + P_{57}' + P_{12} + P_{18} + P_{21} + P_{22} + P_{26}.$$

d) The values that cannot be expressed as sums of two or three other values are

P_{ij} for i = 1,2,3 ; j = 2,5,8

P_{ij} for i = 1,2 ; j = 1,4

and P_{17}, P_{26}, P_{69}.

e) In several cases, alternate values in groups 1 or 2 plus a third value from some other group add to a value in group 6. These cases are:

$P_{21} + P_{23} + P_{54} = P_{67}$
$P_{23} + P_{25} + P_{18} = P_{66}$
$P_{25} + P_{27} + P_{48} = P_{61}$
$P_{27} + P_{29} + P_{12} = P_{63}$

$P_{14} + P_{16} + P_{42} = P_{66}$
$P_{15} + P_{17} + P_{55} = P_{63}$
$P_{15} + P_{17} + P_{35} = P_{66}$

Similar sums add to values in groups 3-5. These are:

$P_{13} + P_{15} + P_{35} = P_{41}$; $P_{13} + P_{15} + P_{55} = P_{54}$

$P_{15} + P_{17} + P_{52} = P_{43}$; $P_{15} + P_{17} + P_{43} = P_{36}$

$P_{16} + P_{18} + P_{51} = P_{36}$

f) The majority of double and triple sums involve at least one value from groups 1,2, or 5'. This is probably because the values in these groups are generally smaller in magnitude. The distribution of pendant values is:

	0-500	501-1000	1001-1500	1501-2500	≥2501
G1,G2,G5'	18	1	1	-	-
G3, G4	3	9	2	3	1
G5,G6	-	5	5	4	3

The sums that are most unlikely to be fortuitous are those involving no group 1,2 or 5' values as summands. They are

among the values larger in magnitude (all ≥1001) and all are in Part II. They are:

$$P_{38} = P_{35} + P_{53} + P_{66} = P_{31} + P_{47} + P_{61}$$

$$P_{49} = P_{41} + P_{46} + P_{52} \; ; \; P_{51}' = P_{52} + P_{55}$$

$$P_{56} = P_{37} + P_{48} + P_{53} = P_{46} + P_{61} + P_{68}$$

$$P_{57} = P_{53} + P_{62} = P_{37} + P_{68}$$

$$P_{58} = P_{32} + P_{51}' + P_{64}$$

$$P_{64} = P_{48} + P_{61} \; ; \; P_{65} = P_{35} + P_{51} \; ; \; P_{67} = P_{51} + P_{62}$$