PHIL633 Vagueness—Delia Graff

Claim: *Modus Ponens* is valid to degree $.5$.

Proof:

$$\frac{\begin{array}{c}[A] \\ [A \supset B]\end{array}}{[B]}$$

When $[A] = .5$ and $[B] = 0$, $[A \supset B] = 1 - ([A] - [B]) = .5$ (by Machina's semantic for the conditional). So in this case, the drop from the minimum value of the premises to the value of the conclusion is $.5$—in other words, the degree of validity of modus ponens is no more than $1 - .5$. (To get degree of validity, substract greatest possible drop from 1.)

So to show that *modus ponens* is valid to degree $.5$, we need to show that no matter what the values of A and B are, the *greatest* possible drop from the minimum value of the premises to the value of the conclusion is $.5$—that is, we need to show that no matter what the values of A and B are, $(\min\{[A], [A \supset B]\} - [B]) \leq .5$.

Case 1: $[B] \geq [A]$.

- In this case, $[A \supset B] = 1$ (by Machina's semantics for the conditional).
- So $[A] \leq [A \supset B]$ (since $0 \leq [A] \leq 1$ for any sentence A).
- So $(\min\{[A], [A \supset B]\} - [B]) = [A] - [B]$.
- But $[A] - [B] \leq 0$ since $[B] \geq [A]$.

Case 2: $[B] < [A]$. In this case, $[A \supset B] = 1 - ([A] - [B])$ (by Machina's semantics).

Case a: $\min\{[A], [A \supset B]\} = [A]$.

- So $[A] \leq 1 - ([A] - [B])$.
- But $[A] - [B] \leq [A]$ since $0 \leq [B]$.
- So $[A] - [B] \leq 1 - ([A] - [B])$ (by transitivity of $\leq$).
- So $2([A] - [B]) \leq 1$ (add $[A] - [B]$ to both sides).
- So $[A] - [B]$ (=drop) $\leq .5$ (divide both sides by 2).

Case b: $\min\{[A], [A \supset B]\} = [A \supset B] = 1 - ([A] - [B])$.

- So $1 - ([A] - [B]) \leq [A]$.
- So $1 + [B] \leq 2[A]$ (add $[A]$ to both sides).
- So $.5 + \frac{[B]}{2} \leq [A]$ (divide both sides by 2).
- So $1 - [A] \leq .5 - \frac{[B]}{2}$
(add $.5$ to each side; subtract $[A]$ from each side; subtract $\frac{[B]}{2}$ from each side).
- But $.5 - \frac{[B]}{2} \leq .5$ (since $\frac{[B]}{2} \geq 0$, since $[B] \geq 0$).
- So $1 - [A]$ (=drop) $\leq .5$.