Econ 620

Assumptions

1. $E(y \mid X) = X\beta \Rightarrow y = X\beta + \varepsilon$
2. X is a non-stochastic $(N \times k)$ matrix with rank k.
3. $E(\varepsilon) = \mathbf{0}$
4. $E(\varepsilon\varepsilon') = \sigma^2 I$
5. $\varepsilon \sim N\left(\mathbf{0}, \sigma^2 I\right)$

What can we derive from the assumptions?

- Under (1) and (2);
$$\widehat{\beta} = (X'X)^{-1} X'y$$
- Under (1) - (3);
$$E\left(\widehat{\beta}\right) = \beta$$
- Under (1) - (4);
$$Var\left(\widehat{\beta}\right) = \sigma^2 (X'X)^{-1}$$
- Under (1) - (4), $\widehat{\beta}$ is BLUE. ($Gauss - Markov\ Theorem$)
- We can estimate σ^2 with
$$s^2 = \frac{1}{N-k} e'e$$
where $e'e = y'y - \widehat{\beta}' X'X\widehat{\beta}$.
-
$$E\left(s^2\right) = \sigma^2$$
- $e = y - X\widehat{\beta} = My$ where $M = I - X (X'X)^{-1} X'$
- M is symmetric, idempotent, positive semi-definite with rank $(N-k)$.
- The coefficient of determination, R^2, is defined as
$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{e'e}{y'Ay}$$
- Under (1) - (5);
$$\widehat{\beta} \sim \left(\beta, \sigma^2 (X'X)^{-1}\right)$$
$$\frac{(N-k) s^2}{\sigma^2} = \frac{e'e}{\sigma^2} \sim \chi^2 (N-k)$$
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$$F = \frac{\left(R\widehat{\beta} - r\right)' \left[R (X'X)^{-1} R'\right]^{-1} \left(R\widehat{\beta} - r\right)/q}{e'e/(N-k)}$$
$$= \frac{\left(R\widehat{\beta} - r\right)' \left[s^2 R (X'X)^{-1} R'\right]^{-1} \left(R\widehat{\beta} - r\right)}{q} \sim F(q, N-k)$$

- $$F=\frac{\left(e_R'e_R-e_U'e_U\right)/q}{e_U'e_U/\left(N-k\right)}\sim F\left(q,N-k\right)$$

The least squares in mean deviation form

- If we decompose the regression as
$$\begin{aligned}y&=X\beta+\varepsilon\\&=\mathbf{1}\beta_1+X_2\beta_2+\varepsilon\end{aligned}$$

- $$\widehat{\beta}_2=\left(X_2'AX_2\right)^{-1}X_2'Ay$$

- $$Var\left(\widehat{\beta}_2\right)=\sigma^2\left(X_2'AX_2\right)^{-1}$$

- We can estimate σ^2 with
$$s^2=\frac{e'e}{\left(N-k\right)}$$
where $e'e=y'Ay-\widehat{\beta}_2'X_2'AX_2\widehat{\beta}_2=y'Ay-\widehat{\beta}_2'X_2'Ay$

- $$R^2=\frac{\widehat{\beta}_2'X_2'AX_2\widehat{\beta}_2}{y'Ay}=\frac{\widehat{\beta}_2'X_2'Ay}{y'Ay}=1-\frac{e'e}{y'Ay}$$

Constrained least squares estimator

- Under the constraint,
$$E\left(b\right)=E\left(\widehat{\beta}+\left(X'X\right)^{-1}R'\left[R\left(X'X\right)^{-1}R'\right]^{-1}\left(r-R\widehat{\beta}\right)\right)=\beta$$

- Under the constraint,
$$Var\left(b\right)=\sigma^2\left[\left(X'X\right)^{-1}-\left(X'X\right)^{-1}R'\left[R\left(X'X\right)^{-1}R'\right]^{-1}R\left(X'X\right)^{-1}\right]\leq Var\left(\widehat{\beta}\right)$$

Specification error

- Exclusion of relevant regressors results in the biased estimator of the included variables.
- Inclusion of irrelevant regressors does not affect the Unbiasedness of the relevant regressors but we lose some efficiency.