Rules: You may use books, notes, or calculators (why?) but not collusion. Enjoy!

1. (30pts Warmup). You devise a clever economic theory leading to a simple regression, which you fit: $y = \hat{\alpha}_1 + \hat{\beta}_1 x$. Your fit is good (high R_1^2, big t-statistic, t_1). Later that evening you have a flash of inspiration: perhaps economics is all wrong, agents are irrational, equilibria don't exist, etc. Perhaps you also have some doubt about whether you derived your equation correctly. Consequently, you fit $x = \hat{\alpha}_2 + \hat{\beta}_2 y$, again finding satisfactory results (high R_2^2 and t_2) and confirming your doubts. The next morning you think this through. What are the relationships between: a) R_1^2 and R_2^2? b) $\hat{\beta}_1$ and $\hat{\beta}_2$? c) t_1 and t_2?

2. (50pts) You have a sample from a Bernoulli process, that is a sample of n observations d_i, where $d_i=1$ if the observation is a success, $d_i=0$ otherwise. The probability of a success is θ, which is the parameter you wish to estimate. Thus, the distribution of d_i is $\theta^d(1-\theta)^{1-d}$. a) What are the loglikelihood function, the score, the information, and the expected information? b) What is the maximum likelihood estimator and its approximate (asymptotic) variance? c) Describe how you would test $H_0: \theta=\theta_0$ vs $H_A: \theta \neq \theta_0$ where $\theta_0 \in (0,1)$? d) (harder) How would you test $H_0: \theta=0$? e) for your test in d, calculate the error probabilities $Pr(Reject|\theta=0)$ and $Pr(Fail\ to\ Reject|\theta=1/2)$ for a sample of size n=1.

3. (20pts) Suppose $X \sim N(0,1)$ (standard normal). a) what is the distribution of X^2? Suppose $Plim(X_n-X) = 0$. b) What is the asymptotic distribution of X_n? c) Suppose $Y_n = X_n^2$. What is the asymptotic distribution of Y_n?