You may use books, notes, calculators (?) or prayer (silent: we are a Federal contractor) but not collusion. Questions count equally. Don't panic. Good luck. Happy St. Patrick's day!

1. Warmup. Consider ther regressions E $y = x_1\beta_1 + x_2\beta_2 + x_3\beta_3$ and $Ey = z_1\alpha_1 + z_2\alpha_2 + z_3\alpha_3$ where $z_1 = x_1 + x_2$, $z_2 = x_1 - x_2$ and $z_3 = x_1 + x_2 + x_3$. Exactly how is β_{LS} related to α_{LS}, the least squares estimators?

2. Mechanics. You begin with the regression model $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$. For mysterious reasons, you are mainly interested in β_2. Let $M_1 = I - X_1(X_1'X_1)^{-1}X_1'$ and let $P_1 = I - M_1$. Your RA, providing cover, estimates the following regressions:

   a) $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$
   b) $P_1y = X_2\beta_2 + \varepsilon$
   c) $P_1y = P_1X_2\beta_2 + \varepsilon$
   d) $M_1y = X_2\beta_2 + \varepsilon$
   e) $y = M_1X_2\beta_2 + \varepsilon$
   f) $M_1y = M_1X_2\beta_2 + \varepsilon$
   g) $M_1y = X_1\beta_1 + M_1X_2\beta_2 + \varepsilon$
   h) $M_1y = M_1X_1\beta_1 + M_1X_2\beta_2 + \varepsilon$

giving you a number of estimates of β_2. How many different estimates are there? How are they related?

3. You estimate a regression model based on n observations and wish to predict the vector of the next 10 observations. Under the usual LS assumptions, what is the covariance matrix of prediction errors? Now suppose you estimated the original model by GLS with a known (!!!) covariance matrix Ω. Further, you know the covariance matrix of the next 10 observations is V and the covariance with the sample you used for estimation is W. What are the predicted values of the next 10 observations? What is the covariance matrix of prediction errors?

4. Consider the regression model with a peculiar trend: $y_t = \beta_1 + \beta_2(1/t) + \varepsilon_t$ with ε_t iid normal with mean zero and variance one. Is the LS estimator for β_2 consistent? What is the exact sampling distribution of the LS estimator? What can you say about the asymptotic distribution? Hint: $\Sigma_1^\infty (1/t)^2 = \pi^2/6$.