Economics 620 Spring, 2004	Midterm	Prof. N. M. Kiefer TA Hwan-sik Choi

You may use one sheet of formulas but not books, etc. Do not collude. Explain your answers briefly (i.e., do not just guess yes or no). All questions (a,b, etc.) count equally. Don't panic. Good luck!

1. (Warmup)
   b.) a.) Prove that for Nx1 real vectors x and y, $x'x = 1$ and $y'y=1$ together imply $1 >=|x'y|$. Hint: consider the vector x-y … . (>= means greater than or equal to).
   c.) Use your result from a. to prove the Cauchy-Schwartz inequality: for arbitrary real vectors x and y, $(x'x)(y'y) >=(x'y)^2$.
2. Suppose you are interested in testing the simple hypothesis H_0: p=1/4 against the alternative H_1: p=3/4 where the observations are independent binomial draws ($f(x) = p^x(1-p)^{(1-x)}$ for x in {0,1}). Let α = Pr(H_0 rejected | H_0 true) and β = Pr(H_0 not rejected| H_1 is true). Suppose you have one observation.
   a.) What is the best test with α = ¼?
   b.) What is the β associated with this test?
   c.) Graph the set of obtainable α, β pairs for tests based on one observation.
   d.) (harder) Graph the set of obtainable α, β pairs for tests based on two observations. (even harder) Interpret briefly but accurately.

3. You have N pairs of points (y_i,x_i) and model their relationship linearly as $Ey_i=\alpha+\beta x_i$. Having forgotten the OLS formulas (now useless on the allowed formula sheet), you reason that a line can be fit by two points, so you arrange the x_i in order and calculate the estimators $\alpha*$ and $\beta*$ by solving the 2 equations $y_1 = \alpha+\beta x_1$ and $y_N=\alpha+\beta x_N$ for values of (α,β).
   a.) Are these estimators unbiased? b.) What is their variance? c.) Are they consistent?
   In view of your results, you decide to calculate sample means for the first and second halves of the sample (luckily N is even). Let $y(1) = (2/N)\Sigma_{i=1}^{N/2}y_i$, $y(2) = (2/N)\Sigma_{i=N/2+1}^{N}y_i$, and similarly for the x. You calculate estimators $\alpha**$ and $\beta**$ by solving $y(1) = \alpha+\beta x(1)$ and $y(2)=\alpha+\beta x(2)$.
   d.) Are these estimators unbiased? e.) What is their variance? f.) Are they consistent? g.) Are they BLUE?