Questions from Previous Final Exams

1. (1996) In the non-linear regression model;

$$y_i = g_i(\theta) + \varepsilon_i \qquad i = 1,2,\cdots,N$$

with ε_i independent with mean zero and variance σ^2. The condition;

$$\lim_{N\to\infty} \sum_{i=1}^{N} (g_i(\theta) - g_i(\theta'))^2 = \infty$$

for $\theta' \neq \theta$ is necessary and sufficient for the existence of a consistent estimator of θ. Interpret this condition in the context of the linear regression model.

2. (1996) The ML estimates of θ, a (2×1) vector of parameters, are $(6,2)'$ and their asymptotic normal distribution has variance matrix;

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

It turns out that the really interesting parameter is;

$$\gamma = \theta_1 + \frac{1}{2}\theta_2^2$$

What is the ML estimate of γ, and what is asymptotic variance? How would you test the hypothesis that $\gamma = 6$?

3. (1996) Consider the system;

$$y_1 = \alpha_1 + \beta_1 y_2 + \varepsilon_1$$

$$y_2 = \alpha_2 + \beta_2 x + \varepsilon_2$$

with $E\varepsilon_k^2 = \sigma_k^2$ for $k = 1,2$ and $E\varepsilon_1\varepsilon_2 = \sigma_{12}$. Identify briefly whether these equations are identified and how the parameters might be estimated. Now suppose that you have reason to believe $\sigma_{12} = 0$. How might you estimate the parameters under this assumption? In particular, consider the properties of the OLS estimator. How might you test the restriction that $\sigma_{12} = 0$?

4. (1997) Consider the trinomial distribution;

$$P[y = 0] = p_0, P[y = 1] = p_1, P[y = 2] = p_2 = 1 - p_0 - p_1$$

   a. What are the mean and variance of y?
   b. What are the MLE for p_0, p_1 and p_2 given a sample of N draws from this distribution?
   c. What is the asymptotic distribution of the MLE?
   d. Entropy, a measure of the expected information value of a draw from a distribution, is defined as

$$R = -p_0 \ln p_0 - p_1 \ln p_1 - p_2 \ln p_2$$

   Find a good estimator for R and give its asymptotic distribution.

5. Suppose

$$y_t = \alpha + \beta t + \varepsilon_t$$

$$x_t = \gamma t + u_t$$

where $t = 1,2,\cdots,T$ and $E(\varepsilon_t) = E(u_t) = E(\varepsilon_t u_t) = 0$. You regress y on x and a constant, getting a slope coefficient b. Find plim$b$. In response to a criticism, you expand your model to;

$$y_t = \lambda + \delta t + \eta x_t + \xi_t$$

and fit again. Find plim$\hat{\delta}$ and plim$\hat{\eta}$.

6. Consider the model;

$$y_i = X_i\beta + \varepsilon_i \qquad i = 1, 2, \cdots, N$$

here X_i is $(1 \times K)$. We assume that

$$E(\varepsilon_i) = 0$$

$$Var(\varepsilon_i) = \begin{cases} 1 \text{ with probability } \frac{1}{2} \\ 99 \text{ with probability } \frac{1}{2} \end{cases}$$

That is, about $\frac{1}{2}$ of the observations have variance 1 and $\frac{1}{2}$ have 99 and you do not know which are which.

a. Is $\widehat{\beta}_{OLS}$ unbiased? BLUE? Consistent?
b. What is the asymptotic distribution of $\widehat{\beta}_{OLS}$?
c. Now, suppose you acquire information on which observations have variance 1 and which 99. For simplicity, assume the rows of X are the same for the 2 variance groups, so we can write

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \beta + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}$$

with $Var(\varepsilon_1) = I_{\frac{N}{2}}$ and $Var(\varepsilon_2) = 99 I_{\frac{N}{2}}$. Can you use this information to make a better estimator? What is it? Compare the asymptotic variance of your new estimator with that of $\widehat{\beta}_{OLS}$.