Economics 620 Prof. N.M. Kiefer
Spring 2002 TA Fernando Grosz

Midterm

Answer all questions (if possible). You may use books, notes, luck, etc. (but not collusion). Weights are (20,20,40,20). Enjoy!

1. (warmup) For the model $y_i = \alpha + \varepsilon_i$ with $E\varepsilon_i = 0$, $E\varepsilon_i\varepsilon_j = 0$ for $i \neq j$ and $=\sigma^2 x_i$ for $i=j$ where x_i are observed positive scalars, find the best linear unbiased estimator for α and give its variance.

2. Your colleague is studying unemployment durations using the exponential model $f(t) = \gamma \exp(-\gamma t)$, where t is a duration and γ is the parameter to be estimated. With a sample of N observations $\{t_i\}$, find the MLE for γ. Give an expression for the asymptotic variance of the MLE. It turns out that your colleague is really interested in the mean duration. What is the mean of this distribution? What is the MLE for the mean duration? What is its approximate (that is, asymptotic) variance? Compare with the "usual" estimator of the variance of the sample mean (namely, s^2/N).

3. You are assisting your faculty advisor on a research project, and you have fit the model $Ey = X\beta$ by OLS. He suggests addition of another variable z, with coefficient δ. You enthusiastically suggest 3 methods of estimating δ,

   a. δ^ from regressing y on X and z
   b. δ* from regressing e on z
   c. δ^+ from regressing e on X and z,

   where $e = y - X\beta$^ is the residual vector from your first regression. You conjecture that these are the same. Your advisor acknowledges that this is plausible and agrees to look at your proof. Prove or disprove.

4. Consider the equicorrelated model, with covariance matrix $V_N = I_N + \alpha 1_N 1_N'$, where 1_N is the N-vector of ones. We saw in class that one of the eigenvectors is a constant vector (ie. proportional to 1_N). Calculate the corresponding eigenvalue. Calculate also the rest of the eigenvalues. A friend points out that your calculation shows "there can be arbitrary positive correlation, but not much negative correlation in large economies." Does this make any sense at all? Interpret in terms of your calculation. [Those interested in finance can worry about the portfolio implications].