

# Economics 620, Lecture 1: Empirical Modeling: A Classy Examples

Nicholas M. Kiefer

Cornell University

- Mincer's model of schooling, experience and earnings
- Develops empirical specification from theory of human capital accumulation
- Goal: Understanding the cross-section distribution of income
- Interaction of data and assumptions

# Assumptions

- $S$  years schooling  $\Rightarrow$  earnings  $E(S)$  (with no other investments)
- PDV of lifetime earnings is equated across identical individuals
- Years at work,  $T$ , is independent of  $S$
- Comments?

$$V(S) = \int_S^R E(S, t) e^{-rt} dt$$

- $R$  = retirement time,  $E(S, t)$  is earnings at  $t$
- $E(S, t) = E(S, t') = E(S)$   
 $V(S) = E(S)(e^{-rS} - e^{-rR})/r$
- $V(S) = V(S') = V$
- $R = S + T$

# More Implications

$$\begin{aligned} rV &= E(S)(e^{-rS} - e^{-rS}e^{-rT}) \\ &= E(0)(1 - e^{-rT}) \\ \Rightarrow E(S) &= E(0)e^{rS} \end{aligned}$$

or

$$\ln E(S) = \ln E(0) + rS$$

Nice clue to skewed income distribution: A symmetric distribution of  $S$  can imply a skewed distribution of income (h.c. investment  $\Rightarrow$  skew)

# Post-School Investments

- Important since earnings are not constant after schooling is over
- Distinguish actual earnings  $Y$ ; potential  $E$
- Assume: workers devote fraction  $k$  of time to investment,  $1 - k$  to market work
- $Y = (1 - k)E$  (workers pay for training)
- Suppose the return on investment is  $p$ , so investment of  $kE$  today yields  $pkE$  in all future periods

- Potential earnings growth

$$\partial E(S, t) / \partial t = p k(t) E(S, t)$$

$$\Rightarrow \ln E(S, t) = \ln E(0) + rS + p \int_0^t k(u) du$$

To proceed, we assume the investment function  $k(t)$ . Assume

$$k(t) = k(1 - t/t^*) \text{ for } t < t^*, \text{ } 0 \text{ for } t > t^*.$$

Comments?

# A little more work

$$\ln E(S, t) = \ln E(0) + rS + pkt - (pk/2t^*)t^2$$

- The log of earnings is linear in schooling and quadratic in experience
- Glitch: This equation is for  $E$ , not  $Y$ ; these differ if  $t < t^*$
- $\ln Y(S, t) = \ln E(S, t) + \ln(1 - k(t))$
- $= \ln E(S, t) + \ln(1 - k + kt/t^*)$
- Approximate the second term by a quadratic (good?)



# Finally, The Empirical Specification

$$\ln Y(S, t) = \beta_1 + \beta_2 S + \beta_3 t + \beta_4 t^2$$

$\beta_2$  is the rate of return to schooling

- $t$  is experience - often not measured. Assume continuous post-schooling employment. Then  $t = A - S$ , where  $A$  is age
- **The interpretation of the coefficients depends on the model!!**

- Mincer's model:

$$\ln Y(S, t) = \beta_1 + \beta_2 S + \beta_3(A - S) + \beta_4(A - S)^2$$

- Suppose instead you fit the model

$$\ln Y(S, t) = \beta'_1 + \beta'_2 S + \beta'_3 A + \beta'_4 A^2$$

- $\beta'_2 = \beta_2$ ?
- Difference is whether age or experience is held constant.

# Fit to 1960 Census Data

- Annual earnings, white nonfarm nonstudent men, 31K obs

$$\ln Y = 6.2 + 0.1075S + 0.081t - 0.0012t^2$$

- $R^2 = 0.285$ , enormous  $t$ -statistics (basically a good fit)
- Rate of return approximately 11%
- This simple economic model explains 28.5% of the variance in cross-sectional earnings

- Focussed on goal - relationship between schooling and earnings in the cross-section.
- Practical matters always at the forefront.
- Model is pushed as far as possible.
- Ignored: many “side issues,” unions, imperfect markets, regulations, other sources of individual heterogeneity, etc.
- Still explains 28.5% of variance in earnings!

# Classy Application 2: The Term Structure of Interest Rates

the pattern of interest rates for bonds of different maturities decompose  
long term rates =  $f()$  of current and expected future short term rates.

$n$ -period bonds in period  $t$  pay a lump sum in  $t+n$ .

one dollar invested at period  $t$  returns  $(1 + R(t, n))^n$  dollars in period  $t+n$ .

$R(t, n)$  as  $n$  varies is the "yield curve" at period  $t$ .

# Forward Rates

Forward rates are agreed to in period  $t$  for one-period bonds purchased in period  $t+j-1$  and redeemed in period  $t+j$  ( $j=1,\dots$ ) with return  $F(t,j)$

Consider selling a two-period bond and buying a three-period bond.

This is the same as buying a forward commitment for a one-period bond purchased in period two. Suppose both bonds have face value \$1. In two periods you will pay out  $(1 + R(t,2))^2$  dollars and in the following period you will receive  $(1 + R(t,3))^3$  dollars.

$$F(t, 3) = (1 + R(t, 3))^3(1 + R(t, 2))^{-2} - 1$$

Why?

The first few are related by

$$1 + R(t, 1) = 1 + F(t, 1)$$

$$(1 + R(t, 2))^2 = (1 + R(t, 1))(1 + F(t, 2))$$

$$(1 + R(t, 3))^3 = (1 + R(t, 1))(1 + F(t, 2))(1 + F(t, 3))$$

# A Simplification for Empirical Work

With simple interest or small rates (logs & Taylor approximation)

$$R(t, 1) = F(t, 1)$$

$$R(t, 2) = (R(t, 1) + F(t, 2))/2$$

$$R(t, 3) = (R(t, 1) + F(t, 2) + F(t, 3))/3$$

Alternatively

$$F(t, j) = jR(t, t + j) - (j - 1)R(t, t + j - 1)$$



Speculators indifferent to risk will enter securities markets, forcing forward rates to expected future spot rates.

Rational expectations implies these are the expectations of the spot rates

$$F(t, j) = E(R(t + j - 1, 1) | R(t, 1), R(t - 1, 1), \dots)$$

Under the assumption that conditional expectations are linear we can write

$$\begin{aligned} E_{t+1}(R, (t + j, 1)) &= E_t(R(t + j, 1)) \\ &\quad + (R(t + 1, 1) - E_t(R(t + 1, 1))) \end{aligned}$$

# Empirical Specification

$$\begin{aligned} E_t(R(t+1, 1)) &= E(R(t+1, 1) | R(t, 1), R(t-1, 1), \dots) \\ &= F(t, 1) \end{aligned}$$

Upon substituting

$$F(t, j) - F(t-1, j+1) = (R(t, 1) - F(t-1, 2))$$

Meiselman estimated equations of the form

$$F(t, j) - F(t-1, j+1) = \alpha_j + \beta_j(R(t, 1) - F(t-1, 2))$$

What are the coefficients?

“Error Learning Model” (Sargent)

Many practical difficulties – bonds pay coupons, etc. Meiselman addressed these carefully and systematically.

For given  $t$ , a plot was made of the yield to maturity versus the term of maturity. Observations are securities.

The constant terms are small and insignificantly different from zero.

First empirical application of rational expectations???