Cornell University
Department of Economics

Econ 620 – Spring 2008
Instructor: Professor Kiefer
TA: Jae Ho Yun

Problem Set #4

(Due: April 8, Tuesday)

1. Suppose that the regression model is;

$$\begin{aligned} y &= X\beta + \varepsilon \\ E(\varepsilon) &= 0, E(\varepsilon\varepsilon') = \sigma^2\Omega \end{aligned}$$

   Assume that Ω is known.

   (a) What is the covariance matrix of the OLS and what is the covariance matrix of the GLS estimators of β?
   (b) What is the covariance matrix of the OLS residual vector, $e = y - X\widehat{\beta}_{OLS}$?
   (c) What is the covariance matrix of the GLS residual vector, $\widetilde{e} = y - X\widehat{\beta}_{GLS}$?
   (d) What is the covariance matrix of the OLS and the GLS residual vectors?

2. Find the autocorrelation function of the following processes. Then, for each process, draw the graph of ACF and guess the shape of PAC.(Refer to the Time Series Forecasting lecture note)

   (a) $X_t = \alpha X_{t-1} + \varepsilon_t$ where $|\rho| < 1$ and $\varepsilon_t \sim i.i.d.(o, \sigma_\varepsilon^2)$
   (b) $Y_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2}$ where $\varepsilon_t \sim i.i.d.(o, \sigma_\varepsilon^2)$

3. The following model is specified:

$$\begin{aligned} y_1 &= \gamma_1 y_2 + \beta_{11}x_1 + \varepsilon_1 \\ y_2 &= \gamma_2 y_1 + \beta_{22}x_2 + \beta_{32}x_3 + \varepsilon_2 \end{aligned}$$

   All variables are in measured in deviations from their means. The sample of 25 observations produces the following matrix of sum of squares and cross products:

	y_1	y_2	x_1	x_2	x_3
y_1	20	6	4	3	5
y_2	6	10	3	6	7
x_1	4	3	5	2	3
x_2	3	6	2	10	8
x_3	5	7	3	8	15

(a) Estimate the two equations by OLS.

(b) Estimate the parameters of the two equations by 2SLS.

4. The following model is specified:

$$
\begin{aligned}
y_1 &= \gamma_1 y_2 + \beta_{11} x_1 + \beta_{12} x_2 + \beta_{13} x_3 + \varepsilon_1 \\
y_2 &= \gamma_2 y_1 + \beta_{21} x_1 + \beta_{22} x_2 + \beta_{23} x_3 + \varepsilon_2
\end{aligned}
$$

The error terms have both expectation zero. We consider only exclusion restrictions. Using the order and rank conditions, verify whether the model is identified under the following restrictions:

(a) $\beta_{12} = \beta_{13} = \beta_{21} = 0$

(b) $\beta_{11} = \beta_{12} = \beta_{13} = 0$

(c) $\beta_{13} = \beta_{22} = 0$

(d) $\beta_{11} = \beta_{12} = 0$

5. (2005 Final) You have a regression model $y_i = \alpha + \beta x_i + \varepsilon_i$ where x is either 0 or 1. In an attempt to simplify your estimation problem, you calculate $\overline{y}_0$ and $\overline{y}_1$, wher these are the sample means corresponding to observations with x=o and x=1 respectively. Then you calculate α^* and β^* by $\overline{y}_0$ and $\overline{y}_1 - \overline{y}_0$. Are your estimators unbiased? Consistent? Efficient(minimum variance unbiased)?