Cornell University
Department of Economics

Econ 620 – Spring 2008
Instructor: Professor Kiefer

Problem Set # 3

(Due: Tuesday, March 3rd in Class)

1. You have a sample from a Bernoulli process, that is a sample of n observations $d_i = 1$ if the observation is a success, $d_i = 0$ otherwise. The probability of a success is θ, which is the parameter you wish to estimate. Thus, the distribution of d_i is $\theta^d \theta^{1-d}$.

   (a) What are the loglikelihood function, the score, the information, and the expected information?

   (b) What is the maximum likelihood estimator and its approximate (asymptotic) variance?

   (c) Describe how you would test $H_0 : \theta = \theta_0$ vs. $H_A : \theta \neq \theta_0$ where $\theta_0 \in (0, 1)$?

2. For the model $y_i = \alpha + \varepsilon_i$ with $E(\varepsilon_i) = 0, E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$ and $= \sigma^2 x_i$ for $i = j$ where x_i are observed positive scalars, find the best linear unbiased estimator for α and give its variance.

3. Consider the classical multiple regression model

   $$\text{Model I: } y = X\beta + \varepsilon = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

   where X_1 is an $N \times k_1$ matrix and X_2 is an $N \times k_2$ matrix with $k_1 + k_2 = k$ and the vector β is partitioned conformably. Now you are given another model such that

   $$\text{Model II: } M_2 y = M_2 X_1 \beta_1 + \varepsilon$$

   where $M_2 = I - X_2(X_2' X_2)^{-1} X_2'$. Show that the least squares estimator for β_1 from Model I is identical to that from Model II.

4. For the standard normal regression model;

   $$y = X\beta + \varepsilon, \ \varepsilon \sim N(0, \sigma^2 I)$$

   (a) Write down the log-likelihood function. And find MLE for β and σ^2.

   (b) Find the asymptotic distribution of MLE.

(c) Prove that

$$E(-\frac{\partial^2 LogL}{\partial\beta\partial\beta'}) = E([\frac{\partial LogL}{\partial\beta}][\frac{\partial LogL}{\partial\beta'}]')$$

5. Consider the following regression model;

$$y = X\beta + \varepsilon,\ \varepsilon \sim N(0, \sigma^2 I)$$

with $E(\varepsilon) = 0, E(\varepsilon\varepsilon') = \sigma^2 I$. Three potential linear estimatiors for β are

$$\begin{aligned}
\widehat{\beta} &= (X'X)^{-1}X'y \\
\widetilde{\beta} &= \widehat{\beta} + N^{-1}1 \\
\overline{\beta} &= \widehat{\beta} + N^{-\frac{1}{2}}1
\end{aligned}$$

where 1 is a $k \times 1$ vector of ones.

(a) Which of these are unbiased?

(b) Which are consistent?

(c) What are the asymptotic distributions of $\sqrt{N}(\widehat{\beta} - \beta), \sqrt{N}(\widetilde{\beta} - \beta)$, and $\sqrt{N}(\overline{\beta} - \beta)$?

6. Suppose $x_i, i = 1, 2, \cdots$ is a sequence of independent random variables where each x_i is uniformly distributed with density

$$f(x_i) = 1_{1[0 \leq x_i < 1]} \text{ for all } i$$

(a) Find $p\lim \frac{1}{n}\sum_{i=1}^{n} x_i, p\lim \frac{1}{n}\sum_{i=1}^{n} x_i^2$ and $p\lim \frac{1}{n}\sum_{i=1}^{n} x_i^3$

(b) Suppose $x_i's$ are as above and $y_i = x_i^2 + \varepsilon_i$ with ε_i independent of x_i and $E(\varepsilon_i) = 0, Var(\varepsilon_i) = \sigma^2$. You run the regression $Ey_i = \alpha + \beta x_i$. Find $p\lim \widehat{\alpha}$ and $p\lim \widehat{\beta}$ where $\widehat{\alpha}$ and $\widehat{\beta}$ are the least squares estimators.