Cornell University
Department of Economics

Econ 620 – Spring 2008
Instructor: Professor Kiefer

Problem Set # 2

(Due : Tuesday, February 26th)

1. Suppose X$\sim$N(0,1) (standard normal). a) what is the distribution of X^2? Suppose $p\lim(X_n - X) = 0$. b) What is the asymptotic distribution of X$_n$? c) Suppose $Y_n = X_n^2$. What is the asymptotic distribution of Y_n?

2. Let Z = [y X] where the ith row of X is (1 x$_i$) and

$$Z'Z = \begin{pmatrix} 760 & 30 & 1300 \\ 30 & 31 & 0 \\ 1300 & 0 & 2480 \end{pmatrix}$$

   Naturally, you regress y on X. Calculate the slop coefficient in the regression.

$$Ey = \alpha + \beta x$$

   What is the mean of x? What is the sample size? What is the R^2? Calculate the OLS estimator and the F-statistics for testing $\alpha = \beta = 0$. Upon thinking about the problem (for the first time), you decide that the right regression is the regression of x on y: $Ex = \alpha^* + \beta^* y$. Calculate the slope coefficient, R^2 and the t-statistics(for β^*). Explain.

3. You begin with the regression model $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$. For mysterious reason, you are mainly interested in β_2. Let $M_1 = I - X_1(X_1'X_1)^{-1}X_1'$ and $P_1 = I - M_1$. Your RA, providing cover, estimating the following regressions:

   (a) $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$
   (b) $P_1 y = X_2\beta_2 + \varepsilon$
   (c) $P_1 y = P_1 X_2\beta_2 + \varepsilon$
   (d) $M_1 y = X_2\beta_2 + \varepsilon$
   (e) $y = M_1 X_2\beta_2 + \varepsilon$
   (f) $M_1 y = M_1 X_2\beta_2 + \varepsilon$
   (g) $M_1 y = X_1\beta_1 + M_1 X_2\beta_2 + \varepsilon$
   (h) $M_1 y = M_1 X_1\beta_1 + M_1 X_2\beta_2 + \varepsilon$

giving you a number of estimates of β_2. What models (among b)~h)) give you the same $\widehat{\beta_2}$ as that of a)? Explain.

4. Suppose that you have two independent unbiased estimators, say b_1 and b_2 of the same parameter β with different variances, say V_1 and V_2 respectively. What linear combination $b = c_1 b_1 + c_2 b_2$ is the minimum variance unbiased estimator of β?

5. Stochastic Regressors:

   Consider the model $y = X\beta + \varepsilon$. Let b be the OLS estimator of β. We will only relax the assumption that X is the matrix of constants; now the regressors are random variables, but uncorrelated with the error term.

   (a) Find the unconditional expectation of b. Is b unbiased?
   Hint: We know that $b = \beta + (X'X)^{-1}X'\varepsilon$. First, find the conditional expectation of b (this is straightforward: $E[b \mid X] = \beta + (X'X)^{-1}X'E[\varepsilon \mid X]$). Then, use the law of iterated expectations.

   (b) Find the unconditional variance of b and show that $Var(b) = \sigma^2 E[(X'X)^{-1}]$.
   Hint: Use the decomposition
   $Var(b) = E_X[Var(b \mid X)] + Var_X E[b \mid X]$.

6. Suppose that

$$
\begin{aligned}
X_n &= 3 - \frac{1}{n^2} \\
Y_n &= \sqrt{n}\frac{\overline{Z_n}}{\sigma}
\end{aligned}
$$

   where $\overline{Z_n} = \frac{1}{n}\sum_{i=1}^{n} Z_i$ and Z_i's are i.i.d. with mean zero and variance σ^2.Find the limiting distribution of

   a) $X_n + Y_n$ b) $X_n Y_n$ c) Y_n^2

7. Consider the following regression model;

$$y = X\beta + \varepsilon$$

   Assume that

$$p\lim \frac{X'X}{N} = Q \text{ where } Q \text{ is positive definite}$$

   and $E(\varepsilon) = 0, E(\varepsilon\varepsilon') = \sigma^2 I$. We also assume that X is non-stochastic.

   (a) Prove that $\widehat{\beta} \xrightarrow{p} \beta$.

   (b) Find the asymptotic distribution of $\sqrt{N}(\widehat{\beta} - \beta)$.

   (c) Prove that $p\lim S^2 = \sigma^2$ where $S^2 = \frac{e'e}{N-k}$