Cornell University
Department of Economics

Econ 620 – Spring 2008
Instructor: Professor Kiefer

Problem Set # 1

(Due : February 19)

1. Suppose the conditional expectation function is $E(y_i \mid x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$. However, I run a regression on constant and x_i only. What is the conditional expectation of $\widehat{\beta}_1$, that is, $E(\widehat{\beta}_1 \mid x)$? Is this unbiased for β_1? In addition, compute the conditional variance of $\widehat{\beta}_1, Var(\widehat{\beta}_1 \mid x)$.

2. (Final 2003) Let $y \sim N(0,1)$. Consider the bivariate distribution of y and z = a*y for some scalar a. What is the correlation between y and z? What is the support of the joint distribution of y and z(the set in $\mathbb{R}^2$ with positive density)? Give a sketch of this set in $\mathbb{R}^2$. Now consider the random variable $x = y^2$. What is the correlation between y and x? What is the support of the joint distribution of y and x? Sketch. Interprete briefly and intelligently, with attention to the interpretation of goodness of fit statistics in regression.

3. (Midterm 2001) Consider the simple regression model with no intercept $y_i = \beta x_i + \varepsilon_i (i = 1, 2)$ and suppose that the true value of β is 1 and the values of x realized in your sample are $x_1 = 1$ and $x_2 = 2$. The distribution of ε is given by $P(\varepsilon = -1) = P(\varepsilon = 1) = \frac{1}{2}$, and the ε_i are independent.

   (a) Does this model satisfy the requirement for OLS(ordinary least wquares) to be BLUE? (You don't need to provide a proof here)
   (b) Calculate the exact distribution of the OLS estimator.
   (c) Consider the alternative estimator $\beta^* = \frac{\Sigma y}{\Sigma x}$ and calculate its exact distribution. Is it unbiased?
   (d) Compare the exact variances of the OLS estimator and β^*. Which one is smaller?

4. Suppose that we have the following information:

$$n = 22,\ \sum_i x_i = 220,\ \sum_i y_i = 440,$$
$$\sum_i x_i^2 = 2260, \sum_i y_i^2 = 8900, \sum_i x_i y_i = 4430,$$

   (a) Compute the OLS estimators of α and β in the model
$$y_i = \alpha + \beta x_i + \varepsilon_i$$
   where $E(\varepsilon_i) = 0, V(\varepsilon_i) = \sigma^2$,and $E(\varepsilon_i \varepsilon_j) = 0$ when $i \neq j$.

(b) Compute the R^2.

(c) Assume now that the errors are normally distributed. Test the following hypothesis at the 5% significance level:
$H_0 : \beta = 0,\ H_A : \beta \neq 0$

(d) Test the following hypothesis at the 10% significance level:
$H_0 : \alpha - \beta = 10,\ H_A : \alpha - \beta \neq 10$

5. (University of Michigan, 1981) Let the regression equation be partitioned as $y = X_1\beta_1 +\ X_2\beta_2 + \varepsilon$. Let b_1 and b_2 be the ususal least-squares estimator. Suppose that $E(\varepsilon) = X_1\gamma$, that is , the mean vector of the disturbances is a linear combination of some of the regressors. Prove that b_1 is biased but b_2 is unbiased.

6. (Midterm 2006) You devise a clever economic theory leading to a simple regression, which you fit: $y = \widehat{\alpha}_1 + \widehat{\beta}_1 x$. Your fit is good(high R_1^2,big t-statistics, t_1). Later that evening you have a flash of inspiration: perhaps you also have some doubt about whether you derived your equaiton correctly. Consequently, you fit $x = \widehat{\alpha}_2 + \widehat{\beta}_2 y$, again finding satisfactory results((high R_2^2 and t_2) and confirming your doubts. The next morning you think this is through. What are the relationships between: a) R_1^2 and R_2^2? b) $\widehat{\beta}_1$ and $\widehat{\beta}_2$? c) t_1 and t_2?