Cornell University
Department of Economics

Econ 620 - Spring 2004
Instructor: Prof. Kiefer

Problem set # 7

1. A model is specified as

$$Y_t = \delta Y_{t-1} + u_t, |\delta| < 1$$
$$u_t = \rho u_{t-1} + \varepsilon_t, |\rho| < 1$$

where $\varepsilon_t \sim i.i.d. \left(0, \sigma_\varepsilon^2\right)$. Let

$$\widehat{\rho} = \frac{\sum_{t=2}^T \widehat{u}_t \widehat{u}_{t-1}}{\sum_{t=2}^T \widehat{u}_{t-1}^2}$$

where $\widehat{u}_t = Y_t - \widehat{\delta} Y_{t-1}$ and $\widehat{\delta}$ be the OLS estimator of δ.

a) Show that plim $\widehat{\delta} = \delta + \frac{\rho\left(1-\delta^2\right)}{(1-\delta\rho)\left[1+\frac{2\delta\rho^2}{1-\delta\rho}\right]}$ (i.e., is not a consistent estimator of δ).

b) Show that $\widehat{\rho}$ is not a consistent estimator of ρ.

c)Find the probability limit of the Durbin-Watson test statistic:

$$d = \frac{\sum_{t=2}^T \left(\widehat{u}_t - \widehat{u}_{t-1}\right)^2}{\sum_{t=1}^T \widehat{u}_t^2}$$

2. Let z_t be a mean zero iid Gaussian time series with variance σ^2. Let c be a constant. Determine for the following processes whether they are or are not covariance stationary. If a process is covariance stationary, compute its mean and autocovariance function.

a) $\mathrm{y}_t = z_1 \cos(ct) + z_2 \sin(ct)$

b) $\mathrm{y}_t = z_t z_{t-1}$

c) $\mathrm{y}_t = z_t \cos(ct) + z_{t-1} \sin(ct)$