

Cornell University
Department of Economics

Econ 620 - Spring 2004
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Problem set # 6

1. Suppose you observe (y_i, x_i) for $i = 1, 2, \dots, N$ and you assume

$$f(y_i | x_i; \alpha, \beta) = \gamma_i \exp(-\gamma_i y_i)$$

where $\gamma_i = \exp[\alpha + \beta x_i]$. What is the MLE for (α, β) and its asymptotic distribution? Find the asymptotic distribution of

$$\hat{\delta} = \hat{\alpha}_{ML} \exp(\hat{\beta}_{ML}).$$

Hint: for the MLE for (α, β) give the system to be solved. Do not try to give an expression (an analytic solution), since it must be solved numerically.

2. Suppose that the regression model is;

$$y = X\beta + \varepsilon$$

$$E(\varepsilon) = 0, E(\varepsilon\varepsilon') = \sigma^2\Omega$$

Assume that Ω is known.

- What is the covariance matrix of the OLS and what is the covariance matrix of the GLS estimators of β ?
 - What is the covariance matrix of the OLS residual vector, $e = y - X\hat{\beta}_{OLS}$?
 - What is the covariance matrix of the GLS residual vector, $\tilde{e} = y - X\hat{\beta}_{GLS}$?
 - What is the covariance matrix of the OLS and the GLS residual vectors?
3. A model is specified as

$$Y_t = \delta Y_{t-1} + u_t, |\delta| < 1$$

$$u_t = \varepsilon_t + \alpha \varepsilon_{t-1}, |\alpha| < 1$$

where $\varepsilon_t \sim i.i.d. (0, \sigma_\varepsilon^2)$. Show that;

- $\text{plim } \hat{\delta} = \delta + \frac{\phi(1-\delta^2)}{1+2\delta\phi}$ where $\hat{\delta} = \frac{\sum_{t=2}^T Y_t Y_{t-1}}{\sum_{t=2}^T Y_{t-1}^2}$ and $\phi = \frac{\alpha}{1+\alpha^2}$.
- $\text{plim } \frac{1}{T} \sum_{t=1}^T \hat{u}_t^2 = \sigma_\varepsilon^2 [1 + \alpha(\alpha - \alpha^*)]$ where $\alpha^* = \frac{\phi(1-\delta^2)}{1+2\delta\phi}$ and $\hat{u}_t = Y_t - \hat{\delta}Y_{t-1}$.

4. In the rational lag model

$$y_t = \frac{3L}{1 - 0.9L + 0.2L^2}x_t + u_t$$

determine;

(a) The total multiplier (b) The mean lag (c) The coefficients of x_{t-j} for $j = 0, 1, 2, 3$.