Cornell University
Department of Economics

Econ 620 - Spring 2004
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Problem set # 5

1. Find the autocorrelation function of the following processes.
   (a) $X_t = \alpha X_{t-1} + \varepsilon_t$ where $|\rho| < 1$ and $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$
   (b) $Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$ where $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$
   (c) $Z_t = \rho Z_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$where $|\rho| < 1$ and $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$

2. Suppose that we have the following regression model;

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 Y_{t-1} + u_t$$

   where X_t is non-stochastic. Further, we have the following error structure;

$$u_t = \rho u_{t-1} + \varepsilon_t$$

   where $|\rho| < 1$ and $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$.
   (a) Prove that

$$\text{plim}\widehat{\beta}_3 \neq \beta_3$$

   where $\widehat{\beta}_3$ is the OLS estimator of β_3.
   (b) How can you obtain a consistent estimator for β_3?

3. Consider estimation of σ^2 in the generalized linear regression model. There is a fundamental ambiguity in regard to this parameter as it is merely a scaling of $E\left(\varepsilon\varepsilon'\right) = \sigma^2\Omega$. Since both components are unknown, σ^2 cannot be estimable until some scaling of Ω is assumed to remove the indeterminacy. The most convenient assumption is that

$$tr\left(\Omega\right) = N$$

   The classical regression model in which $\Omega = I$ is one such case, so this provides a useful benchmark. Now, consider the estimator $s^2 = e'e/\left(N-k\right)$, where e is the vector of the OLS residuals.
   (a) Prove that

$$E\left(s^2\right) = \frac{N\sigma^2}{(N-k)} - \frac{\sigma^2 tr\left[\left(\frac{X'X}{N-k}\right)^{-1}\left(\frac{X'\Omega X}{N-k}\right)\right]}{(N-k)}$$

(b) Prove that if

$$\begin{aligned}
\text{plim}\frac{X'X}{N} &= Q \text{ where } Q \text{ is positive definite} \\
\text{plim}\frac{X'\varepsilon}{N} &= \mathbf{0} \\
\text{plim}\frac{X'\Omega X}{N} &= L \text{ where } L \text{ is positive definite}
\end{aligned}$$

then,

$$\lim_{N\to\infty} E\left(s^2\right) = \sigma^2$$

(c) To consider the issue of consistency, prove that

$$\text{plim}s^2 = \text{plim}\left(\frac{1}{N-k}\right)\sum_{i=1}^{N}\varepsilon_i^2$$

Under what conditions is $\text{plim}s^2 = \sigma^2$?

4. What are the cases in which the seemingly unrelated regression estimator(SURE) is equivalent to the OLS estimator? Prove your claim.

5. a) Expand the rational lag model:

$$y_t = \tfrac{0.6+2L}{1-0.6L+0.5L^2}x_t + \varepsilon_t$$

What are the coefficients on $x_{t-1}, x_{t-2}, x_{t-3}$ and x_{t-4}?

b) Suppose that the model in part a) were specified as

$$y_t = \alpha + \tfrac{\beta+\gamma L}{1-\delta L-\eta L^2}x_t + \varepsilon_t$$

How can the parameters be estimated? Is OLS consistent?