Cornell University
Department of Economics

Econ 620 - Spring 2004
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Problem set # 4

1)
Consider the classical multiple regression model

$$\text{Model I; } y = X\beta + \varepsilon = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

where X_1 is an $(N \times k_1)$ matrix and X_2 is an $(N \times k_2)$ matrix with $k_1 + k_2 = k$ and the vector β is partitioned conformably.. Now, you are given another model such that

$$\text{Model II; } M_2 y = M_2 X_1 \beta_1 + \epsilon$$

where $M_2 = I - X_2 \left(X_2' X_2\right)^{-1} X_2'$. Show that the least squares estimator for β_1 from Model I is identical to that from Model II.

2)

The production function is

$$\log Q = \beta_0 + \beta_1 \log L + \beta_2 \log K + \varepsilon$$

and you are primarily interested in the elasticity of output with respect to labor. Unfortunately, you have no capital data. You proceed to regress $\log Q$ on $\log L$ alone. What if anything can be said about the relationship between the slope coefficient on $\log L$ you estimated and the parameter of primary interest? Is there any case in which the dropping the regressor, $\log K$, does not affect the result of regression?

3)

A generalization of the Cobb-Douglas production function is the translog production function;

$$\log Q = \beta_1 + \beta_2 \log L + \beta_3 \log K + \beta_4 \frac{(\log L)^2}{2} + \beta_5 \frac{(\log K)^2}{2} + \beta_6 \log L \log K + \varepsilon$$

(a) Show that the condition for the constant returns to scale is

$$\frac{\partial \log Q}{\partial \log L} + \frac{\partial \log Q}{\partial \log K} = 1$$

(b) What restrictions on the coefficients produce constant returns to scale? How would you estimate the restricted model? How would you test the hypothesis of the constant returns to scale?

4)

Consider the following regression model;

$$y = X\beta + \varepsilon$$

with $E(\varepsilon) = 0, E(\varepsilon\varepsilon') = \sigma^2 I$. Three potential linear estimators for β are

$$\begin{aligned}
\widehat{\beta} &= (X'X)^{-1} X'y \\
\widetilde{\beta} &= \widehat{\beta} + N^{-1}\mathbf{1} \\
\overline{\beta} &= \widehat{\beta} + N^{-\frac{1}{2}}\mathbf{1}
\end{aligned}$$

where $\mathbf{1}$ is a $(k \times 1)$ vector of ones.

a) Which of these are unbiased?
b)Which are consistent?

c)What are the asymptotic distributions of $\sqrt{N}\left(\widehat{\beta} - \beta\right)$, $\sqrt{N}\left(\widetilde{\beta} - \beta\right)$, and $\sqrt{N}\left(\overline{\beta} - \beta\right)$?

5)

(Midterm,1997) Suppose x_i , i = 1, 2, ... is a sequence of independent random variables where each x_i is uniformly distributed with density

$$f(x_i) = 1_{\mathbf{1}[0 \leq x_i < 1]} \text{ for all } i$$

(a) Find $\text{plim}\frac{1}{n}\sum_{i=1}^{n} x_i$,$\text{plim}\frac{1}{n}\sum_{i=1}^{n} x_i^2$, and $\text{plim}\frac{1}{n}\sum_{i=1}^{n} x_i^3$.

For the rest of the exercise:

Suppose $x_i's$ are as above and $y_i = x_i^2 + \varepsilon_i$ with ε_i independent of x_i and $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$. You run the regression $Ey_i = \alpha + \beta x_i$.

(b) Find plim $\widehat{\alpha}$ and plim $\widehat{\beta}$ where $\widehat{\alpha}$ and $\widehat{\beta}$ are the least squares estimators.

(c) You are really interested in the "average derivative" $E\left(\frac{dy}{dx}\right)$. Comment $\widehat{\beta}$ as an estimator of this quantity.

(d) You now realize that theory requires that $E(y) = 0$ when $x = 0$. Reasoning that it cannot hurt to impose a restriction you know to be true, you run the regression $Ey_i = \gamma x_i$(no intercept). Find plim$\widehat{\gamma}$. Comment on $\widehat{\gamma}$ as an estimator of the average derivative.

6)

For the standard normal regression model;

$$y = X\beta + \varepsilon, \varepsilon \sim N\left(\mathbf{0}, \sigma^2 I\right)$$

(a) Write down the log-likelihood function. And find MLE for β and σ^2.

(b) Find the asymptotic distribution of MLE.

(c) Prove that

$$E\left(-\frac{\partial^2 \log L}{\partial\beta\partial\beta'}\right) = E\left(\left[\frac{\partial \log L}{\partial\beta}\right]\left[\frac{\partial \log L}{\partial\beta}\right]'\right)$$