Cornell University
Department of Economics

Econ 620 - Spring 2003
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Problem set # 2
Due February 14^{th}

1) Find the transformations that will linearize each of these functions:

a) $y = \frac{x}{\alpha x - \beta}$
b) $y = \frac{e^{\alpha + \beta x}}{1 + e^{\alpha + \beta x}}$

2) Suppose that you have two independent unbiased estimators, say b_1 and b_2 of the same parameter β,with different variances, say v_1 and v_2 respectively. What linear combination $b = c_1 b_1 + c_2 b_2$ is the minumum variance unbiased estimator of β?

3) You want to fit the model $y = \alpha + \beta x + \varepsilon$, but you do not have the full data set [y X] = Z. Instead, you only have Z'Z:

$$Z'Z = \begin{bmatrix} 100 & 10 & 25 \\ 10 & 20 & 0 \\ 25 & 0 & 75 \end{bmatrix}$$

Calculate $\hat{\beta}, s^2$ and R^2.

4) Show that in the model $y = X\beta + \varepsilon$, the square of the correlation between y and $\hat{y}$ is equal to R^2. ($where \quad \hat{y} = \hat{\beta}_1 + \hat{\beta}_2 x_2 + ... + \hat{\beta}_k x_k$).

5) Suppose we regress x on y. We want to minimize the sum of squares of the residuals (RSS), but the residuals are now the horizontal distance (instead of the vertical distance), viewed in a graph with x in the horizontal axis. This gives another RSS which will correspond to the RSS of the regression of x on y. So we can regress y on x or x on y , but only one R^2 can be computed for the data set.

Let b_{yx} and b_{xy} be the estimators of the slope in the regression of y on x and x on y respectively.

a) When will $b_{yx} = \frac{1}{b_{xy}}$?

b) Will they always have the same sign?

c) Show that $r^2 = b_{yx}\, b_{xy}$.

d) From "c", it follows that $b_{yx} \leq 1/b_{xy}$ provided $b_{xy} > 0$. That is, viewed in a graph with X in the horizontal axis, the regression of x on y has a higher slope. Check part "c" and that $b_{yx} \leq 1/b_{xy}$ provided $b_{xy} > 0$ for this sample of 200 observations where:

$$\sum x = 11.34 \qquad \sum y = 20.72 \qquad \sum x^2 = 12.16 \qquad \sum y^2 = 84.96 \qquad \sum xy = 22.13$$

6) Stochastic regressors:

Consider the model $y = X\beta + \varepsilon$. Let b be the OLS estimator of β. We will only relax the assumption that X is a matrix of constants; now the regressors are random variables, but uncorrelated with the error term

a) Find the unconditional expectation of b. Is b unbiased?

Hint: we know that b$=\beta + (X'X)^{-1}X'\varepsilon$. First, find the conditional expectation of b (this is straightforward: E[b$|\ X] = \beta + (X'X)^{-1}X'E[\varepsilon \mid X]$). Then, use the law of iterated expectations.

b) Find the unconditional variance of b and show that Var (b)$=\sigma^2 E[(X'X)^{-1}]$.

Hint: use the decomposition $Var(b) = E_X[Var(b \mid X)] + Var_X E[b \mid X]$.