Cornell University
Department of Economics

Econ 620 - Spring 2004
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Problem set # 1

1) Show that $logY = \alpha + \beta \log X + u$ gives the same least squares estimator of β whether the logs are taken to base 10 or to base e. Is this true for the estimator of α? Do your conclusions change if log X is replaced by t (time)?

2) The following exercise is a remark on bivariate normal distributions and normality of its component:

Evaluate the following statement: if true, prove it; if false, show a counterexample: if X and Y are two standard normal random variables, then the joint distribution of (X,Y) is bivariate normal.

Remark : note that we are not saying anything about X and Y being independent. If they were, the statement is true. You have to consider the case of X and Y not necessarily being independent, and do not make them perfectly (positively or negatively) correlated. Also note that if (X,Y) is bivariate normal, then X will be a normal random variable and Y as well (but not necessarily standard normal).

Exercise 2 is difficult and you are allowed to skip it if you cannot come with the proof or a counterexample.

3) (Midterm 2001) Consider the simple regression model with no intercept

$$y_i = \beta x_i + \varepsilon_i \quad i = 1, 2$$

and suppose that the true value of β is 1 and the values of x realized in your sample are $x_1 = 1$ and $x_2 = 2$. The distribution of ε is given by $P(\varepsilon = -1) = P(\varepsilon = 1) = \frac{1}{2}$, and the ε_i are independent.

a) Does this model satisfy the requirements for OLS (ordinary least squares) to be BLUE ? (You do not need to provide a proof here)

b) Calculate the exact distribution of the OLS estimator.

c) Consider the alternative estimator $\beta^* = \frac{\sum y}{\sum x}$ and calculate its exact distribution. Is it unbiased?

d) compare the exact variances of the OLS estimator and β^*. Which one is smaller?

4) Suppose that the have the following information :

$$n = 22, \quad \sum_i x_i = 220, \quad \sum_i y_i = 440,$$
$$\sum_i x_i^2 = 2260, \quad \sum_i y_i^2 = 8900, \quad \sum_i x_i y_i = 4430$$

a) Compute the OLS estimators of α and β in the model

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$$E(\varepsilon_i) = 0, \qquad V(\varepsilon_i) = \sigma^2 \qquad \text{and} \quad E(\varepsilon_i \varepsilon_j) = 0 \quad \text{when } i \neq j.$$

b) Compute the R^2

c) Assume now that the errors are normally distributed. Test the following hypothesis at the 5 % significance level:

$H_0 : \beta = 0 \qquad H_A : \beta \neq 0$

d) Test the following hypothesis at the 10 % significance level:

$H_0 : \alpha - \beta = 10 \qquad H_A : \alpha - \beta \neq 10$

5) (This question is mechanic but worth trying)
Show that the OLS estimator of the intercept in the model

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$E(\varepsilon_i) = 0, \qquad V(\varepsilon_i) = \sigma^2 \qquad$ and $E(\varepsilon_i \varepsilon_j) = 0$ when $i \neq j \qquad$ is BLUE.

Hint: Write $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = \frac{1}{n}\sum_i y_i - \bar{x}\frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \sum_i [\frac{1}{n} - \bar{x}\frac{(x_i - \bar{x})}{\sum_i (x_i - \bar{x})^2}] y_i = \sum_i m_i y_i$ where $m_i = [\frac{1}{n} - \bar{x}\frac{(x_i - \bar{x})}{\sum_i (x_i - \bar{x})^2}]$.