Sample Midterm Problems

Econ 620 Spring 2001

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1. (Midterm, 1997) Consider the least squares residual vector e from the regression of y on X, where $V(y) = \sigma^2 I$. Show that the variance of any element of e, say e_j, is less that or equal to σ^2.

2. (Midterm, 199?) Consider the OLS estimator in the model $y = X\beta + \varepsilon$ with $E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon') = \sigma^2 I$. Let the first column of X consist of ones and let the other regressors be measured in deviations from means. Show that the estimator of the intercept is uncorrelated with the estimators of the slopes.

3. (Midterm, 1996) You have in mind the model

$$Ey = X\beta = X_1\beta_1 + X_2\beta_2,$$
$$V(y \mid X) = \sigma^2 I$$

but you are really only interested in β_1. Consider the following estimators;

(a) (the direct approach) β_1^*, coefficients from a regression of y on X_1

(b) (a transformed model) you decide to get rid of β_2 by projecting everything in sight to the space orthogonal to the column space of X_2(what is the matrix Q of this projection?) resulting in data

$$Q\begin{bmatrix} y & X_1 & X_2 \end{bmatrix} = \begin{bmatrix} Qy & QX_1 & \mathbf{0} \end{bmatrix}$$

and regress Qy on QX_1 giving coefficients $\widetilde{\beta}_1$

(c) (a lazy transformation) you forgot to transform the dependent variable and consider data $\begin{bmatrix} y & QX_1 \end{bmatrix}$; the regression coefficients are $\overline{\beta}_1$.

(d) Finally, you consider the OLS estimator $\widehat{\beta}$ from the regression of y on $\left[\begin{array}{cc} X_1 & X_2 \end{array} \right]$ and select the elements corresponding to $\widehat{\beta}_1$; $\overrightarrow{\beta}_1 = R\widehat{\beta}$ with $R = \left[\begin{array}{cc} \mathbf{1} & \mathbf{0} \end{array} \right]$.

Compare the estimators- find the mean and variance of each estimator and compare them.

4. (Final, 1997) Consider the standard regression model

$$y = X\beta + \varepsilon$$

where y is an $(N \times 1)$ vector, X is an $(N \times k)$ matrix and ε is an $(N \times 1)$ vector. Suppose there were another set of available regressors Z whose dimension is $(N \times k)$. Suppose further that there exists an $(k \times k)$ non-singular matrix G such that $Z = XG$.

Let $\widehat{\beta}$ be the least squares estimator from the regression of Y on X and let $\widehat{\gamma}$ be the least squares estimator from the regression of Y on Z.

(a) Show that $\widehat{\gamma} = G^{-1}\widehat{\beta}$.

(b) Show that $Var\left(\widehat{\gamma}\right) = \left(G^{-1}\right) Var\left(\widehat{\beta}\right)\left(G^{-1}\right)'$.

(c) Show that s^2 from the regression of Y on X is exactly the same as s^2 from the regression of Y on Z.

(d) Suppose you multiplied all of the variables in X by 100 to obtain a new set of regressors Z. What is the G matrix in this case? Suppose Y were regressed on this Z. How would the $t-$statistics of the coefficients estimates compare to the $t-$statistics from the regression of Y on X. So, how does the scaling the X variables affect hypothesis testing?

5. (Final, 1996) You fit the model

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

and you find that the R^2 is O.K., the F is significant, but the $t-$statistics are lousy. β_2 is the coefficient of primary interest. Having read about collinearity, you decide to change the model and regress y on X_1 and X_2^* where X_2^* is the projection of X_2 to the space orthogonal to the space spanned by the columns of X_1. Note that the sets of regressors X_1 and X_2^* are orthogonal.

(a) This projection is obtained by multiplying X_2 by what matrix?

(b) Compare the R^2 and F (for the overall specification) in the new specification to those in the original regression.

(c) Compare the coefficients on X_2^* in the new model to that on X_2 in the original model.

(d) What happens to your t-statistics on the estimates of β_2?

Suggested solutions

1. Recall that

$$\begin{aligned} e &= y - X\widehat{\beta} = y - X\left(X'X\right)^{-1}X'y = \left[I - X\left(X'X\right)^{-1}X'\right]y = My \\ &= M\left(X\beta + \varepsilon\right) = MX\beta + M\varepsilon = M\varepsilon \end{aligned}$$

Then,

$$E\left(e\right) = E\left(M\varepsilon\right) = ME\left(\varepsilon\right) = 0$$

since $M = \left[I - X\left(X'X\right)^{-1}X'\right]$ is non-stochastic. Hence,

$$\begin{aligned} Var\left(e\right) &= E\left[\left(e - E\left(e\right)\right)\left(e - E\left(e\right)\right)'\right] = E\left[ee'\right] \\ &= E\left[M\varepsilon\varepsilon'M'\right] = ME\left[\varepsilon\varepsilon'\right]M = \sigma^2 MIM \\ &= \sigma^2 M \end{aligned}$$

note that M is symmetric and idempotent. The variance matrix of e is an $(N \times N)$ matrix. The variance of e_j is the (j,j) element of the variance matrix, which can be picked up by

$$Var\left(e_j\right) = \sigma^2 M^{jj} = \sigma^2\left[I - X\left(X'X\right)^{-1}X'\right]^{jj} = \sigma^2\left[1 - X_j\left(X'X\right)^{-1}X'^{j}\right]$$

where X_j is the jth row of X and X'^{j} is the jth column of X'. Then,

$$\begin{aligned} Var\left(e_j\right) - \sigma^2 &= \sigma^2\left[1 - X_j\left(X'X\right)^{-1}X'^{j}\right] - \sigma^2 \\ &= -\sigma^2 X_j\left(X'X\right)^{-1}X'^{j} \\ &= -\sigma^2 X_j\left(X'X\right)^{-1}X_j' \leq 0 \end{aligned}$$

since $X_j\left(X'X\right)^{-1}X_j'$ is a quadratic form in $\left(X'X\right)^{-1}$ and we know that $\left(X'X\right)$ is positive semidefinite and hence so is $\left(X'X\right)^{-1}$.

2. What is the operator we use to get mean deviation form? Yes, it is $A = I - \mathbf{1}(\mathbf{1}'\mathbf{1})^{-1}\mathbf{1}'$. Then, the X matrix is now;

$$X = \begin{bmatrix} \mathbf{1} & AX_2 \end{bmatrix}$$

where $\mathbf{1}$ is an $(N \times 1)$ vector of ones and X_2 is an $(N \times (k-1))$ matrix of independent variables except for the constant term. Therefore,

$$\begin{aligned} X'X &= \begin{bmatrix} \mathbf{1}'\mathbf{1} & \mathbf{1}'AX_2 \\ X_2'A\mathbf{1} & X_2'AA'X_2 \end{bmatrix} = \begin{bmatrix} N & \mathbf{0} \\ \mathbf{0} & X_2'AX_2 \end{bmatrix} \\ X'y &= \begin{bmatrix} \mathbf{1}'y \\ X_2'Ay \end{bmatrix} \end{aligned}$$

note that $\mathbf{1}'A = A\mathbf{1} = \mathbf{0}$ and again A is symmetric idempotent. Hence,

$$\begin{aligned} Var\left(\widehat{\beta}\right) &= \sigma^2 (X'X)^{-1} = \sigma^2 \begin{bmatrix} N & \mathbf{0} \\ \mathbf{0} & X_2'AX_2 \end{bmatrix}^{-1} \\ &= \sigma^2 \begin{bmatrix} \frac{1}{N} & \mathbf{0} \\ \mathbf{0} & (X_2'AX_2)^{-1} \end{bmatrix} \end{aligned}$$

We use the fact that $(X'X)$ is block diagonal. The covariance between the intercept and the slope estimator is the off-diagonal term, which is $\mathbf{0}$.

3. (a) $\beta_1^* = (X_1'X_1)^{-1} X_1'y$. Then,

$$\beta^*{}_1 = (X_1'X_1)^{-1} X_1' (X_1\beta_1 + X_2\beta_2 + \varepsilon)$$

$$= \beta_1 + (X_1'X_1)^{-1} X_1'X_2\beta_2 + (X_1'X_1)^{-1} X_1'\varepsilon$$

Hence,

$$\begin{aligned} E(\beta_1^*) = \beta_1 + (X_1'X_1)^{-1} X_1'X_2\beta_2 + (X_1'X_1)^{-1} X_1'E(\varepsilon) \\ = \beta_1 + (X_1'X_1)^{-1} X_1'X_2\beta_2 \end{aligned}$$

This estimator is biased. On the other hand,

$$Var(\beta_1^*) = E\left[(\beta_1^* - E(\beta_1^*))(\beta_1^* - E(\beta_1^*))'\right]$$
$$= E\left[(X_1'X_1)^{-1} X_1'\varepsilon\varepsilon'X_1 (X_1'X_1)^{-1}\right] = \sigma^2 (X_1'X_1)^{-1}$$

(b) The projection matrix which projects into the space orthogonal to the space spanned by the columns of X_2 is given by

$$Q = I - X_2 \left(X_2'X_2\right)^{-1} X_2' = M_{X_2}$$

Therefore,

$$\begin{aligned}
\widetilde{\beta}_1 &= \left(X_1'M_{X_2}'M_{X_2}X_1\right)^{-1}\left(X_1'M_{X_2}'M_{X_2}y\right) \\
&= \left(X_1'M_{X_2}X_1\right)^{-1}\left(X_1'M_{X_2}y\right) = \left(X_1'M_{X_2}X_1\right)^{-1} X_1'M_{X_2}\left(X_1\beta_1 + X_2\beta_2 + \varepsilon\right) \\
&= \beta_1 + \left(X_1'M_{X_2}X_1\right)^{-1} X_1'M_{X_2}\varepsilon
\end{aligned}$$

since $M_{X_2}X_2 = 0$. Then,

$$\begin{aligned}
E\left(\widetilde{\beta}_1\right) &= E\left[\beta_1 + \left(X_1'M_{X_2}X_1\right)^{-1} X_1'M_{X_2}\varepsilon\right] \\
&= \beta_1
\end{aligned}$$

This estimator is unbiased. The variance matrix is given by

$$\begin{aligned}
Var\left(\widetilde{\beta}_1\right) &= E\left[\left(\widetilde{\beta}_1 - E\left(\widetilde{\beta}_1\right)\right)\left(\widetilde{\beta}_1 - E\left(\widetilde{\beta}_1\right)\right)'\right] \\
&= E\left[\left(X_1'M_{X_2}X_1\right)^{-1} X_1'M_{X_2}\varepsilon\varepsilon'M_{X_2}'X_1\left(X_1'M_{X_2}X_1\right)^{-1}\right] \\
&= \sigma^2\left(X_1'M_{X_2}X_1\right)^{-1}
\end{aligned}$$

Which is the smaller, $Var\left(\beta_1^*\right)$ or $Var\left(\widetilde{\beta}_1\right)$? In order to compare the variances, I state a theorem without the proof- it is actually easy to prove and very intuitive-

- $A^{-1} - B^{-1} \geq 0$ if and only if $B - A \geq 0$.

Using the theorem,

$$\begin{aligned}
& Var^{-1}\left(\widetilde{\beta}_1\right) - Var^{-1}\left(\beta_1^*\right) \\
&= \frac{1}{\sigma^2}\left(X_1'M_{X_2}X_1\right) - \frac{1}{\sigma^2}\left(X_1'X_1\right) = \frac{1}{\sigma^2}\left[X_1'\left(M_{X_2} - I\right)X_1\right] \\
&= \frac{1}{\sigma^2}\left[X_1'\left(I - X_2\left(X_2'X_2\right)^{-1}X_2' - I\right)X_1'\right] \\
&= -\frac{1}{\sigma^2}\left[X_1'X_2\left(X_2'X_2\right)^{-1}X_2'X_1'\right] \leq 0
\end{aligned}$$

since $X_2 \left(X_2'X_2\right)^{-1} X_2'$ is a positive semidefinite matrix - remember the minus sign front -. Therefore,

$$Var\left(\widetilde{\beta}_1\right) \geq Var\left(\beta_1^*\right)$$

i.e., β_1^* is biased but has smaller variance than $\widetilde{\beta}_1$.

(c) We find quite an interesting result that the transformation of the dependent variable does not affect the result of the least squares estimator in that

$$\begin{aligned}
\overline{\beta}_1 &= \left(X_1'M_{X_2}'M_{X_2}X_1\right)^{-1}\left(X_1'M_{X_2}'y\right) \\
&= \left(X_1'M_{X_2}X_1\right)^{-1}\left(X_1'M_{X_2}y\right) = \widetilde{\beta}_1
\end{aligned}$$

Therefore, $\overline{\beta}_1$ is unbiased and $Var\left(\overline{\beta}_1\right) = \sigma^2 \left(X_1'M_{X_2}X_1\right)^{-1}$.

(d) $\widehat{\beta}$ is the usual least squares estimator;

$$\widehat{\beta} = \left(X'X\right)^{-1} X'y = \beta + \left(X'X\right)^{-1} X'\varepsilon$$

Hence,

$$\overrightarrow{\beta}_1 = R\widehat{\beta} = R\left[\beta + \left(X'X\right)^{-1} X'\varepsilon\right]$$

Then,

$$\begin{aligned}
E\left(\overrightarrow{\beta}_1\right) &= R\beta + R\left(X'X\right)^{-1} X'E\left(\varepsilon\right) = R\beta \\
&= \begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \beta_1
\end{aligned}$$

This estimator is unbiased. The variance matrix is given by;

$$Var\left(\overrightarrow{\beta}_1\right) = Var\left(R\widehat{\beta}\right) = RVar\left(\widehat{\beta}\right)R' = \sigma^2 R\left(X'X\right)^{-1} R'$$

To figure out $Var\left(\overrightarrow{\beta}_1\right)$ in detail, note that

$$\begin{aligned}
\sigma^2 R\left(X'X\right)^{-1} R' &= \sigma^2 \begin{bmatrix} \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_1'X_1 & X_1'X_2 \\ X_2'X_1 & X_2'X_2 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{1}' \\ \mathbf{0}' \end{bmatrix} \\
&= \sigma^2 \left(X_1'M_{X_2}X_1\right)^{-1}
\end{aligned}$$

Therefore, $\overrightarrow{\beta}_1$ is unbiased and $Var\left(\overrightarrow{\beta}_1\right) = Var\left(\widetilde{\beta}_1\right)$.

4. The key condition is that G is non-singular.

(a) First of all, we know that

$$\begin{aligned}\widehat{\beta} &= (X'X)^{-1} X'y \\ \widehat{\gamma} &= (Z'Z)^{-1} Z'y\end{aligned}$$

Consider the condition that $Z = XG$;

$$\begin{aligned}\widehat{\gamma} = (Z'Z)^{-1} Z'y &= \left[(XG)'(XG)\right]^{-1}(XG)'y \\ = [G'X'XG]^{-1} G'X'y &= G^{-1}(X'X)^{-1}(G')^{-1} G'X'y \\ &= G^{-1}(X'X)^{-1} X'y = G^{-1}\widehat{\beta}\end{aligned}$$

(b)

$$\begin{aligned}Var(\widehat{\gamma}) &= E\left[(\widehat{\gamma} - E(\widehat{\gamma}))(\widehat{\gamma} - E(\widehat{\gamma}))'\right] \\ &= E\left[\left(G^{-1}\widehat{\beta} - E\left(G^{-1}\widehat{\beta}\right)\right)\left(G^{-1}\widehat{\beta} - E\left(G^{-1}\widehat{\beta}\right)\right)'\right] \\ &= E\left[G^{-1}\left(\widehat{\beta} - E\left(\widehat{\beta}\right)\right)\left(\widehat{\beta} - E\left(\widehat{\beta}\right)\right)'\left(G^{-1}\right)'\right] \\ &= \left(G^{-1}\right)E\left[\left(\widehat{\beta} - E\left(\widehat{\beta}\right)\right)\left(\widehat{\beta} - E\left(\widehat{\beta}\right)\right)'\right]\left(G^{-1}\right)' \\ &= \left(G^{-1}\right)Var\left(\widehat{\beta}\right)\left(G^{-1}\right)'\end{aligned}$$

(c)

$$\begin{aligned}s^2_{yZ} = \frac{e'_{yZ}e_{yZ}}{N-k} = \frac{y'M_Z y}{N-k} &= \frac{y'\left[I - Z(Z'Z)^{-1}Z'\right]y}{N-k} \\ &= \frac{y'\left[I - (XG)\left[(XG)'(XG)\right]^{-1}(XG)'\right]y}{N-k} \\ &= \frac{y'\left[I - XG[G'X'XG]^{-1}G'X'\right]y}{N-k} \\ &= \frac{y'\left[I - XGG^{-1}(X'X)^{-1}(G')^{-1}G'X'\right]y}{N-k} \\ = \frac{y'\left[I - X(X'X)^{-1}X'\right]y}{N-k} &= \frac{y'M_X y}{N-k} = \frac{e'_{yX}e_{yX}}{N-k} = s^2_{yX}\end{aligned}$$

(d) Note that

$$X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{k1} \\ 1 & x_{12} & \cdots & x_{k2} \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_{1N} & \cdots & x_{kN} \end{bmatrix}$$

We want to have

$$\begin{aligned} Z &= \begin{bmatrix} 100 & 100x_{11} & \cdots & 100x_{k1} \\ 100 & 100x_{12} & \cdots & 100x_{k2} \\ \cdots & \cdots & \cdots & \cdots \\ 100 & 100x_{1N} & \cdots & 100x_{kN} \end{bmatrix} \\ &= \begin{bmatrix} 1 & x_{11} & \cdots & x_{k1} \\ 1 & x_{12} & \cdots & x_{k2} \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_{1N} & \cdots & x_{kN} \end{bmatrix} \begin{bmatrix} 100 & 0 & \cdots & 0 \\ 0 & 100 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 100 \end{bmatrix} = X \times 100I \end{aligned}$$

Hence, $G = 100I$, which is a $(k \times k)$ non-singular matrix. From (a)

$$\widehat{\gamma} = G^{-1}\widehat{\beta} = (100I)^{-1}\,\widehat{\beta} = \frac{1}{100}\widehat{\beta}$$

On the other hand,

$$\begin{aligned} Var(\widehat{\gamma}) &= \sigma^2 \left(G^{-1}\right)(X'X)^{-1}\left(G^{-1}\right)' \\ &= \sigma^2 (100I)^{-1}(X'X)^{-1}\left[(100I)^{-1}\right]' \\ &= \frac{\sigma^2}{10000}(X'X)^{-1} = \frac{1}{10000} Var\left(\widehat{\beta}\right) \end{aligned}$$

From the argument in (c), we have $s_{yZ}^2 = s_{yX}^2$. Then, the estimate of the variance matrix is

$$\widehat{Var}(\widehat{\gamma}) = \frac{1}{10000}\widehat{Var}\left(\widehat{\beta}\right)$$

Now, consider the t-test for the validity of jth coefficient, $\widehat{\gamma}_j$. Then, the test statistic is;

$$t = \left| \frac{\widehat{\gamma}_j}{\sqrt{\widehat{Var}(\widehat{\gamma}_j)}} \right| = \left| \frac{\frac{1}{100}\widehat{\beta}_j}{\sqrt{\frac{1}{10000}\widehat{Var}\left(\widehat{\beta}_j\right)}} \right| = \left| \frac{\widehat{\beta}_j}{\sqrt{\widehat{Var}\left(\widehat{\beta}_j\right)}} \right|$$

which is exactly the t-statistic for the test of validity of jth element of $\widehat{\beta}_j$ in the regression of y on X. Rescaling of independent variable does not affect the result of $t - test$.

5. (a) The projection is given by $M_{X_1} = \left[I - X_1 \left(X_1' X_1\right)^{-1} X_1'\right]$ so that

$$X_2^* = \left[I - X_1 \left(X_1' X_1\right)^{-1} X_1'\right] X_2 = M_{X_1} X_2$$

(b) Let's write the transformed model as;

$$y = X_1 \gamma_1 + X_2^* \gamma_2 + \epsilon$$

The residual sum of squares from above model is equivalent to that from;

$$M_{X_1} y = M_{X_1} X_2^* \gamma_2 + v$$

Note that

$$\widehat{\gamma}_2 = \left(X_2^{*\prime} M_{X_1} X_2^*\right)^{-1} \left(X_2^{*\prime} M_{X_1} y\right)$$

Hence,

$$\begin{aligned} e_{trans}' e_{trans} &= \left(M_{X_1} y - M_{X_1} X_2^* \widehat{\gamma}_2\right)' \left(M_{X_1} y - M_{X_1} X_2^* \widehat{\gamma}_2\right) \\ &= \left(M_{X_1} y - M_{X_1} X_2^* \widehat{\beta}_2\right)' \left(M_{X_1} y - M_{X_1} X_2^* \widehat{\beta}_2\right) \text{ since } \widehat{\gamma}_2 = \widehat{\beta}_2. \text{ See (c);} \\ &= \left(M_{X_1} y - M_{X_1} M_{X_1} X_2 \widehat{\beta}_2\right)' \left(M_{X_1} y - M_{X_1} M_{X_1} X_2 \widehat{\beta}_2\right) \text{ since } X_2^* = M_{X_1} X_2. \text{ S} \\ &= \left(M_{X_1} y - M_{X_1} X_2 \widehat{\beta}_2\right)' \left(M_{X_1} y - M_{X_1} X_2 \widehat{\beta}_2\right) \text{ since } M_{X_1} \text{ is idempotent.} \\ &= e_{orig}' e_{orig} \end{aligned}$$

where e_{orig} is the residual vector from the regression;

$$M_{X_1} y = M_{X_1} X_2 \beta_2 + u$$

whose residual is identical to that from the original model;

$$y = X_1 \beta_1 + X_2 \beta_2 + \varepsilon$$

Now, recall the definitions of R^2 and the F-statistic for the overall specification;

$$\begin{aligned} R^2 &= 1 - \frac{e'e}{y'Ay} \\ F &= \frac{R^2 / (k-1)}{\left(1 - R^2\right) / (N - k)} \end{aligned}$$

Note that we have identical residual sums of squares from the two specifications and the same dependent variable with the same numbers of observations and regressors. Therefore, R^2 and F do not change.

(c) We compare the two models;

$$\begin{aligned} y &= X_1\beta_1 + X_2\beta_2 + \varepsilon \\ y &= X_1\gamma_1 + X_2^*\gamma_2 + \epsilon \end{aligned}$$

Both β_2 and γ_2 can be estimated after removing the terms involved in X_1 as;

$$\begin{aligned} M_{X_1}y &= M_{X_1}X_2\beta_2 + u \\ M_{X_1}y &= M_{X_1}X_2^*\gamma_2 + v \end{aligned}$$

The least squares estimators are now;

$$\begin{aligned} \widehat{\beta}_2 &= \left(X_2'M_{X_1}X_2\right)^{-1}\left(X_2'M_{X_1}y\right) \\ \widehat{\gamma}_2 &= \left(X_2^{*\prime}M_{X_1}X_2^*\right)^{-1}\left(X_2^{*\prime}M_{X_1}y\right) \end{aligned}$$

However,

$$\begin{aligned} X_2^{*\prime}M_{X_1}X_2^* &= \left(M_{X_1}X_2\right)' M_{X_1}\left(M_{X_1}X_2\right) = X_2'M_{X_1}'M_{X_1}M_{X_1}X_2 = X_2'M_{X_1}X_2 \\ X_2^{*\prime}M_{X_1}y &= \left(M_{X_1}X_2\right)' M_{X_1}y = X_2'M_{X_1}'M_{X_1}y = X_2'M_{X_1}y \end{aligned}$$

Hence,

$$\widehat{\gamma}_2 = \widehat{\beta}_2$$

(d)

$$\begin{aligned} Var\left(\widehat{\beta}_2\right) &= \sigma^2\left(X_2'M_{X_1}X_2\right)^{-1} \\ Var\left(\widehat{\gamma}_2\right) &= \sigma^2\left(X_2^{*\prime}M_{X_1}X_2^*\right)^{-1} = \sigma^2\left(X_2'M_{X_1}X_2\right)^{-1} \text{ See (c)} \end{aligned}$$

Moreover, when we estimate σ^2, we use the estimators;

$$\begin{aligned} s_{orig}^2 &= \frac{e_{orig}'e_{orig}}{N-k} = \frac{e_{trans}'e_{trans}}{N-k} \text{ See (a)} \\ &= s_{trans}^2 \end{aligned}$$

The t-statistics are identical since we have identical estimates for both parameter and its variance matrix.