Economics 620 Midterm Prof. N. M. Kiefer
Spring (?) 2001 TA N. Hashimzade

You may use books, calculators, tarot, etc. but do not collude. Panic is not appropriate. Econometrics is fun! Good luck.

1. (Warmup: 30%) Consider the simple regression model with no intercept

   $$y_i = x_i\beta + \varepsilon_i \quad i = 1,2$$

   and suppose that the true value of β is 1 and the values of x realized in your sample are $x_1=1$, $x_2=2$. The distribution of ε is given by $P(\varepsilon=-1) = P(\varepsilon=1) = ½$ and the ε_i are independent. a) Does this model satisfy the requirements for ols to be BLUE? b) calculate the exact distribution of the ols estimator. c) consider the alternative estimator $\beta^* = \sum y/\sum x$ and calculate its exact distribution. Is β^* unbiased? d) compare the exact variances of β^* and the ols estimator.

2. A crafty empirical colleague is testing an efficiency hypothesis which requires that $\beta=0$ in the following regression model: $y = X\beta + \varepsilon$, where y is Nx1, X NxK, etc. and ε is normally distributed with $E\varepsilon=0$, $V(\varepsilon) = \sigma^2 I$. Assume for simplicity that the data are measured in deviations from means (so there is no intercept). Your colleague comments that he usually finds the score test appropriate for this hypothesis. A colleague hostile to the hypothesis asserts that the Wald test is obviously better. You point out that the tests are asymptotically equivalent and both colleagues give you a funny look.
   a) Calculate the score statistic S, the likelihood ratio test statistic LR, and the Wald statistic W for the hypothesis H_0: $\beta=0$.
   b) Interpret all of these statistics as transformations of R^2.
   c) Establish the inequalities $S \leq LR \leq W$.
   d) Derive the asymptotic distribution of these statistics, without using the normality assumption. Interpret.
   e) Comment intelligently and briefly. What test would you recommend, and why?

Note that you have established c) only for the linear model. The result is not general.