Econ 620
Spring, 1999
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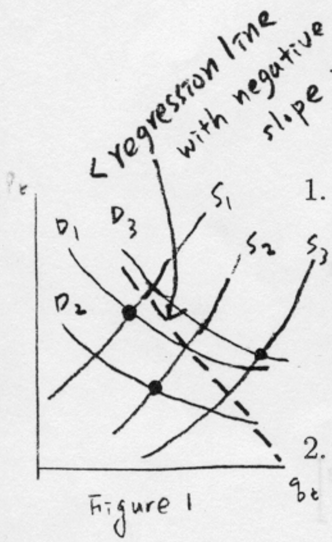
Suggested Solution to the Midterm Exam



1. The pairs $(q_t, p_t)_{t=1}^{T}$ are series of equilibrium at each point jointly determined by demand curve and supply curve. The fact that we have variations in data set indicates that both supply curve and demand move around - see Figure 1. Therefore, it is most likely that the equation my local macroeconomist estimated is neither demand curve nor supply curve.

2. Note that

$$\widehat{\beta} = \frac{\sum_{t=1}^{T}\left(p_t - \overline{p}\right)\left(q_t - \overline{q}\right)}{\sum_{t=1}^{T}\left(p_t - \overline{p}\right)^2} \text{ and } \widehat{\beta}^* = \frac{\sum_{t=1}^{T}\left(p_t - \overline{p}\right)\left(q_t - \overline{q}\right)}{\sum_{t=1}^{T}\left(q_t - \overline{q}\right)^2}$$

Therefore,

$$\widehat{\beta} = \widehat{\beta}^* \Longleftrightarrow \sum_{t=1}^{T}\left(p_t - \overline{p}\right)^2 = \sum_{t=1}^{T}\left(q_t - \overline{q}\right)^2$$

3. Recall that

$$\begin{aligned} t_{\widehat{\beta}} &= \frac{\widehat{\beta}}{s.e.\left(\widehat{\beta}\right)} = \frac{\widehat{\beta}}{\left[\frac{s^2}{\sum_{t=1}^{T}(p_t-\overline{p})^2}\right]^{\frac{1}{2}}} = \frac{\frac{\sum_{t=1}^{T}(p_t-\overline{p})(q_t-\overline{q})}{\sum_{t=1}^{T}(p_t-\overline{p})^2}}{\left[\frac{\sum_{t=1}^{T}e_t^2}{(T-2)\sum_{t=1}^{T}(p_t-\overline{p})^2}\right]^{\frac{1}{2}}} \\ &= \frac{(T-2)^{\frac{1}{2}}\sum_{t=1}^{T}\left(p_t-\overline{p}\right)\left(q_t-\overline{q}\right)}{\left[\sum_{t=1}^{T}e_t^2\sum_{t=1}^{T}\left(p_t-\overline{p}\right)^2\right]^{\frac{1}{2}}} \end{aligned}$$

On the other hand

$$t_{\widehat{\beta}^*} = \frac{\widehat{\beta}^*}{s.e.\left(\widehat{\beta}^*\right)} = \frac{\widehat{\beta}^*}{\left[\frac{s^{*2}}{\sum_{t=1}^T (q_t-\overline{q})^2}\right]^{\frac{1}{2}}} = \frac{\frac{\sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})}{\sum_{t=1}^T (q_t-\overline{q})^2}}{\left[\frac{\sum_{t=1}^T e_t^{*2}}{(T-2)\sum_{t=1}^T (q_t-\overline{q})^2}\right]^{\frac{1}{2}}}$$

$$= \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})}{\left[\sum_{t=1}^T e_t^{*2} \sum_{t=1}^T (q_t-\overline{q})^2\right]^{\frac{1}{2}}}$$

However,

$$\sum_{t=1}^T e_t^2 = \sum_{t=1}^T \left(q_t - \widehat{\alpha} - \widehat{\beta} p_t\right)^2 = \sum_{t=1}^T \left[(q_t-\overline{q})^2 - \widehat{\beta}^2 (p_t-\overline{p})^2\right]$$

$$= \sum_{t=1}^T (q_t-\overline{q})^2 - \widehat{\beta}^2 \sum_{t=1}^T (p_t-\overline{p})^2$$

$$= \sum_{t=1}^T (q_t-\overline{q})^2 - \left[\frac{\sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})}{\sum_{t=1}^T (p_t-\overline{p})^2}\right]^2 \sum_{t=1}^T (p_t-\overline{p})^2$$

$$= \sum_{t=1}^T (q_t-\overline{q})^2 - \frac{\left[\sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})\right]^2}{\sum_{t=1}^T (p_t-\overline{p})^2}$$

$$\sum_{t=1}^T e_t^{*2} = \sum_{t=1}^T \left(p_t - \widehat{\alpha}^* - \widehat{\beta}^* q_t\right)^2 = \sum_{t=1}^T \left[(p_t-\overline{p})^2 - \widehat{\beta}^{*2} (q_t-\overline{q})^2\right]$$

$$= \sum_{t=1}^T (p_t-\overline{p})^2 - \widehat{\beta}^{*2} \sum_{t=1}^T (q_t-\overline{q})^2$$

$$= \sum_{t=1}^T (p_t-\overline{p})^2 - \frac{\left[\sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})\right]^2}{\sum_{t=1}^T (q_t-\overline{q})^2}$$

In sum, we have

$$t_{\widehat{\beta}} = \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})}{\left[\sum_{t=1}^T e_t^2 \sum_{t=1}^T (p_t-\overline{p})^2\right]^{\frac{1}{2}}}$$

$$= \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})}{\left[\left\{\sum_{t=1}^T (q_t-\overline{q})^2 - \frac{\left[\sum_{t=1}^T (p_t-\overline{p})(q_t-\overline{q})\right]^2}{\sum_{t=1}^T (p_t-\overline{p})^2}\right\} \sum_{t=1}^T (p_t-\overline{p})^2\right]^{\frac{1}{2}}}$$

$$= \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})}{\left[\sum_{t=1}^{T} (q_t - \overline{q})^2 \sum_{t=1}^{T} (p_t - \overline{p})^2 - \left[\sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})\right]^2\right]^{\frac{1}{2}}}$$

and

$$\begin{aligned} t_{\widehat{\beta}^*} &= \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})}{\left[\sum_{t=1}^{T} e_t^{*2} \sum_{t=1}^{T} (q_t - \overline{q})^2\right]^{\frac{1}{2}}} \\ &= \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})}{\left[\left\{\sum_{t=1}^{T} (p_t - \overline{p})^2 - \frac{\left[\sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})\right]^2}{\sum_{t=1}^{T} (q_t - \overline{q})^2}\right\} \sum_{t=1}^{T} (q_t - \overline{q})^2\right]^{\frac{1}{2}}} \\ &= \frac{(T-2)^{\frac{1}{2}} \sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})}{\left[\sum_{t=1}^{T} (p_t - \overline{p})^2 \sum_{t=1}^{T} (q_t - \overline{q})^2 - \left[\sum_{t=1}^{T} (p_t - \overline{p})(q_t - \overline{q})\right]^2\right]^{\frac{1}{2}}} \end{aligned}$$

i.e. it is always true that $t_{\widehat{\beta}} = t_{\widehat{\beta}^*}$.

4. Solving the demand - supply system for price yields;

$$\begin{aligned} q_t^S &= q_t^D \\ \gamma + \delta p_{t-1} + v_t &= \alpha + \beta p_t + \varepsilon_t \\ p_t &= \frac{1}{\beta}(\gamma - \alpha) + \frac{\delta}{\beta} p_{t-1} + \frac{1}{\beta}(v_t - \varepsilon_t) \end{aligned}$$

Hence, we should have

$$\xi = \frac{1}{\beta}(\gamma - \alpha), \theta = \frac{\delta}{\beta}$$

We can determine the parameters in VAR model once we have estimates for the parameters in system wide approach, not vice versa.

5. We know that

$$\begin{bmatrix} \widehat{\alpha} \\ \widehat{\beta} \\ \widehat{\gamma} \\ \widehat{\delta} \end{bmatrix} \xrightarrow{d} N\left(\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix}, \begin{bmatrix} a_{\alpha\alpha} & a_{\alpha\beta} & a_{\alpha\gamma} & a_{\alpha\delta} \\ a_{\alpha\beta} & a_{\beta\beta} & a_{\beta\gamma} & a_{\beta\delta} \\ a_{\alpha\gamma} & a_{\beta\gamma} & a_{\gamma\gamma} & a_{\gamma\delta} \\ a_{\alpha\delta} & a_{\beta\delta} & a_{\gamma\delta} & a_{\delta\delta} \end{bmatrix}\right)$$

By the invariance property of ML estimator, we know that

$$\widehat{\theta} = \frac{\widehat{\delta}}{\widehat{\beta}}$$

Moreover, δ-method tells me that

$$\widehat{\theta} \xrightarrow{d} N\left(\theta, Var\left(\widehat{\theta}\right)\right)$$

where

$$\begin{aligned} Var\left(\widehat{\theta}\right) &= \begin{bmatrix} \frac{\partial\theta}{\partial\alpha} & \frac{\partial\theta}{\partial\beta} & \frac{\partial\theta}{\partial\gamma} & \frac{\partial\theta}{\partial\delta} \end{bmatrix} \begin{bmatrix} a_{\alpha\alpha} & a_{\alpha\beta} & a_{\alpha\gamma} & a_{\alpha\delta} \\ a_{\alpha\beta} & a_{\beta\beta} & a_{\beta\gamma} & a_{\beta\delta} \\ a_{\alpha\gamma} & a_{\beta\gamma} & a_{\gamma\gamma} & a_{\gamma\delta} \\ a_{\alpha\delta} & a_{\beta\delta} & a_{\gamma\delta} & a_{\delta\delta} \end{bmatrix} \begin{bmatrix} \frac{\partial\theta}{\partial\alpha} \\ \frac{\partial\theta}{\partial\beta} \\ \frac{\partial\theta}{\partial\gamma} \\ \frac{\partial\theta}{\partial\delta} \end{bmatrix} \\ &= \begin{bmatrix} 0 & -\frac{\delta}{\beta^2} & 0 & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} a_{\alpha\alpha} & a_{\alpha\beta} & a_{\alpha\gamma} & a_{\alpha\delta} \\ a_{\alpha\beta} & a_{\beta\beta} & a_{\beta\gamma} & a_{\beta\delta} \\ a_{\alpha\gamma} & a_{\beta\gamma} & a_{\gamma\gamma} & a_{\gamma\delta} \\ a_{\alpha\delta} & a_{\beta\delta} & a_{\gamma\delta} & a_{\delta\delta} \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{\delta}{\beta^2} \\ 0 \\ \frac{1}{\beta} \end{bmatrix} \\ &= a_{\beta\beta}\frac{\delta^2}{\beta^4} - 2a_{\beta\delta}\frac{\delta}{\beta^3} + a_{\delta\delta}\frac{1}{\beta^2} \end{aligned}$$

We can consistently estimate the variance of $\widehat{\theta}$ with

$$\widehat{Var}\left(\widehat{\theta}\right) = a_{\beta\beta}\frac{\widehat{\delta}^2}{\widehat{\beta}^4} - 2a_{\beta\delta}\frac{\widehat{\delta}}{\widehat{\beta}^3} + a_{\delta\delta}\frac{1}{\widehat{\beta}^2}$$

6. Note that $\theta = \frac{\delta}{\beta}$. Basic economic theory -say, Econ 101- dictates that

$$\delta > 0 \text{ and } \beta < 0 \text{ - why?}$$

Therefore, the negative correlation of the price series is totally congruent to the prediction of (structural) economic model