

MIDTERM EXAM

You may use books, notes, calculators (buy why?) and prayer (silent: we are a Federal contractor) but do not collude. Good luck!

Your local macroeconomist (this is not Cornell) shows up at your office for advice on modelling the hog sector. He has regressed q_t , the log of the quantity of hogs in time t , on p_t , the log of the price of hogs, obtaining

$$q_t = \hat{\alpha} + \hat{\beta}p_t + e_t$$

Noting that $\hat{\beta} < 0$, and consulting a local theorist, he concludes that this is a demand function.

- a. Comment intelligently on this conclusion.

Next, he reasons that, with the demand side given, prices are determined by the quantity supplied. Since supply and demand are equal in equilibrium, he estimates

$$p_t = \hat{\alpha}^* + \hat{\beta}^*q_t + e_t^*$$

If these relationships were exact, i.e., $e_t = 0 \forall t$, then $\hat{\beta}^* = 1/\hat{\beta}$. But $e_t \neq 0$ and $\hat{\beta}^* \neq 1/\hat{\beta}$.

- b. Explain the relation between $\hat{\beta}^*$ and $\hat{\beta}$. When are they equal? For extra credit, comment snidely on the state of macroeconomics.

Surprisingly, your colleague has noted that the t-statistic for $\hat{\beta} = 0$ is the same as the t-statistic for $\hat{\beta}^* = 0$. You assume he has made a calculation error and send him away.

- c. What is the relation between these t-statistics? When are they equal?

Feeling chagrined about (c) you offer your colleague a supply equation

$$q_t^S = \gamma + \delta p_{t-1} + v_t$$

which together with the demand equation $q_t^D = \alpha + \beta p_t + \varepsilon_t$ and the equilibrium condition $q_t^S = q_t^D$ forms a complete system. Using a system estimation method (we haven't covered these yet) you obtain maximum likelihood estimates $(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta})$ and their asymptotic covariance matrix A . It turns out that your colleague is really interested only in hog prices, as input to the financial sector, and not in quantities. Having read a competitor's paper, he decides to use what he calls a VAR model, namely

$$p_t = \xi + \theta p_{t-1} + v_t$$

- d. What is the relation between this new "VAR" approach and your "system wide" approach?
- e. Give an estimate of θ and the associated variance of your estimator as functions of $(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta})$ and A .

(Notation: let A have elements a_{ij} and A^{-1} elements a^{ij} for $i, j \in \{\alpha, \beta, \gamma, \delta\}$.)

Finally, your colleague is a little concerned that his estimate of θ is negative. In his experience, "most economic time series are positively autocorrelated".

- f. Use the system to give an argument about the expected sign of θ (and turn concern into either satisfaction or panic).