Economics 620
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# Midterm

You may use books, notes, calculators intuition and math, but not collusion. Good luck!

1. (Warmup) You wish to fit the model $y = X\beta + \epsilon$, but you do not have the full data set $[y\ X] = Z$. Instead you have only $Z'Z$

$$Z'Z = \begin{bmatrix} 100 & 10 & 25 \\ 10 & 20 & 0 \\ 25 & 0 & 75 \end{bmatrix}$$

Calculate $\hat{\beta}$, $s^2$ and $R^2$. Is there anything to be gained by seeing the full dataset?

2. You are interested in fitting the regression model

$$y_i = x_i'\beta + \epsilon_i$$

with $x_i'$ $(1 \times 2) = (1\ 1)$ in N/2 observations (Group 1) $x_i' = (1\ -1)$ in the other N/2 observations (Group 2). Although ols will work you design an alternative estimator $\tilde{\beta}$ by noting that

$$* \qquad \bar{y}(1) = \beta_1 + \beta_2 + \bar{\epsilon}(1)$$

$$\bar{y}(2) = \beta_1 - \beta_2 + \bar{\epsilon}(2)$$

where $\bar{y}(j)$ is the sample mean of the $y_i$ in group j, etc. Since $\bar{\epsilon}(j)$, has expectation zero, you solve the 2 linear equations * with the $\bar{\epsilon}$ set to zero to obtain $\tilde{\beta}$. Give a formula for $\tilde{\beta}$. Is $\tilde{\beta}$ unbiased? What is its sampling variance? (Assume $\epsilon_i$ are independent, mean zero, variance $\sigma^2$). Is $\tilde{\beta}$ consistent? What is the variance of the ols estimator? Explain.

**OVER PLEASE**

3. In another simple regression

$$y_i = \alpha + x_i\beta + \epsilon_i$$

with $x_i$ scalar, you do not observe $x_i$ but rather only a badly measured version $z_i = x_i + \nu_i$ (a "proxy"). So you regress $y_i$ on $z_i$ and obtain a slope parameter $\hat{\beta}$. Assume that $\nu_i$ is independent of $x_i$ and $\epsilon_i$. Show that

$$\text{plim } \hat{\beta} < \beta.$$

Explain (briefly).

Now you observe $w_i$ which is correlated with $x_i$ but not $\nu_i$ or $\epsilon_i$. You multiply through and regress $w_i y_i$ on $w_i$ and $w_i z_i$. Find the plim of the coefficient of $w_i z_i$. Explain.